

## DECOMPOSITION OF COMPLETE MULTIGRAPHS INTO STARS AND CYCLES

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### Abstract

Let  $k$  be a positive integer,  $S_k$  and  $C_k$  denote, respectively, a star and a cycle of  $k$  edges.  $\lambda K_n$  is the usual notation for the complete multigraph on  $n$  vertices and in which every edge is taken  $\lambda$  times. In this paper, we investigate necessary and sufficient conditions for the existence of the decomposition of  $\lambda K_n$  into edges disjoint of stars  $S_k$ 's and cycles  $C_k$ 's.

**Keywords:** graph decomposition, complete multigraph, stars, cycles.

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### REFERENCES

- [1] A.A. Abueida and M. Daven, *Multidesigns for graph-pairs of order 4 and 5*, Graphs Combin. **19** (2003) 433–447.  
doi:10.1007/s00373-003-0530-3
- [2] A.A. Abueida and M. Daven, *Multidecompositions of the complete graph*, Ars Combin. **72** (2004) 17–22.
- [3] A.A. Abueida and T. O’Neil, *Multidecomposition of  $\lambda K_m$  into small cycles and claws*, Bull. Inst. Combin. Appl. **49** (2007) 32–40.
- [4] A.A. Abueida and C. Lian, *On the decompositions of complete graphs into cycles and stars on the same number of edges*, Discuss. Math. Graph Theory **34** (2014) 113–125.  
doi:10.7151/dmgt.1719

- [5] B. Alspach and H. Gavlas, *Cycle decompositions of  $K_n$  and  $K_n - I$* , J. Combin. Theory, Ser. B **81** (2001) 77–99.  
doi:10.1006/jctb.2000.1996
- [6] D. Bryant, D. Horsley, B. Maenhaut and B.R. Smith, *Cycle decompositions of complete multigraphs*, J. Combin. Des. **19** (2011) 42–69.  
doi:10.1002/jcd.20263
- [7] V. Chitra and A. Muthusamy, *Symmetric Hamilton cycle decompositions of complete multigraphs*, Discuss. Math. Graph Theory **33** (2013) 695–707.  
doi:10.7151/dmgt.1687
- [8] S. Cichacz, *Decomposition of complete bipartite digraphs and even complete bipartite multigraphs into closed trails*, Discuss. Math. Graph Theory **27** (2007) 241–249.  
doi:10.7151/dmgt.1358
- [9] H.-C. Lee and J.-J. Lin, *Decomposition of the complete bipartite graph with a 1-factor removed into cycles and stars*, Discrete Math. **313** (2013) 2354–2358.  
doi:10.1016/j.disc.2013.06.014
- [10] Z. Liang and J. Guo, *Decomposition of complete multigraphs into crown graphs*, J. Appl. Math. Comput. **32** (2010) 507–517.  
doi:10.1007/s12190-009-0267-0
- [11] H.M. Priyadharsini and A. Muthusamy,  *$(G_m, H_m)$ -multifactorization of  $\lambda K_m$* , J. Combin. Math. Combin. Comput. **69** (2009) 145–150.
- [12] M. Šajna, *Cycle decompositions III: Complete graphs and fixed length cycles*, J. Combin. Des. **10** (2002) 27–78.  
doi:10.1002/jcd.1027
- [13] T.-W. Shyu, *Decompositions of complete graphs into paths and cycles*, Ars Combin. **97** (2010) 257–270.
- [14] T.-W. Shyu, *Decomposition of complete graphs into paths of length three and triangles*, Ars Combin. **107** (2012) 209–224.
- [15] T.-W. Shyu, *Decomposition of complete graphs into cycles and stars*, Graphs Combin. **29** (2013) 301–313.  
doi:10.1007/s00373-011-1105-3
- [16] T.-W. Shyu, *Decomposition of complete bipartite graphs into paths and stars with same number of edges*, Discrete Math. **313** (2013) 865–871.  
doi:10.1016/j.disc.2012.12.020
- [17] D. Sotteau, *Decomposition of  $K_{m,n}$  ( $K_{m,n}^{(*)}$ ) into cycles (circuits) of length  $2k$* , J. Combin. Theory, Ser. B **30** (1981) 75–81.  
doi:10.1016/0095-8956(81)90093-9
- [18] M. Tarsi, *Decomposition of complete multigraphs into stars*, Discrete Math. **26** (1979) 273–278.  
doi:10.1016/0012-365X(79)90034-7

- [19] M. Tarsi, *Decomposition of a complete multigraph into simple paths: Nonbalanced handcuffed designs*, J. Combin. Theory, Ser. A **34** (1983) 60–70.  
doi:10.1016/0097-3165(83)90040-7
- [20] R.M. Wilson, *Decomposition of complete graphs into subgraphs isomorphic to a given graph*, in: Proceedings of the 5th British Combinatorial Conference, Util. Math., Winnipeg, Congr. Numer. **15** (1976) 647–659.
- [21] S. Yamamoto, H. Ikeda, S. Shige-eda, K. Ushio and N. Hamada, *On claw-decomposition of complete graphs and complete bigraphs*, Hiroshima Math. J. **5** (1975) 33–42.

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