RP 148

# APPLYING THE VISUAL DOUBLE-MODULATION TYPE RADIO RANGE TO THE AIRWAYS

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ABSTRACT

This paper deals with methods for aligning the courses of the visual radio range with the fixed airways. It has previously been shown that the courses of the aural radio range may be shifted by the use of a vertical wire antenna in conjunction with the transmitting loop antennas or by varying the relative power in the two antennas. These methods are, in part, applicable to the visual system. In the aural system the goniometer primaries are excited alternately. This permits independent consideration of the field patterns due to the primaries. In the visual system this is not the case, as both goniometer primaries are excited all the time. Two cases present themselves, the condition when the currents in the primaries are in time phase and the condition when they are in quadrature time phase. The former condition results in two beacon courses which are 180° apart and can not be shifted from this relationship. The latter condition yields four beacon courses. A mathematical analysis is made of this case, and the amounts of angular variation possible using several methods of attack are tabulated.

A method of obtaining small amounts of shift by an adjustment of the receiving equipment aboard the airplanes is also described; one of the reeds is shunted by a suitable resistance in order that the reeds will vibrate equally when on one side of the equisignal zone. This method permits of great flexibility in securing a desired course and is suitable only for employment with the visual system. Sample calculations are made for actual airway routes to demonstrate the several methods of attack.

CONTENTS



## I. INTRODUCTION

Methods for aligning the courses of the aural type radio range—that is, directive radiobeacon—with the fixed airways are described in a paper by Kear and Jackson.[1] Extensions of these methods may be used to make the visual double-modulation type radio range equally flexible. This paper describes a number of circuit arrangements, the application of which makes possible the use of a single visual type radio range for serving two, three, or four courses radiating from a given airport at arbitrary angles with each other.

The procedure of aligning the courses of the visual type radio range with the fixed airways is necessarily somewhat different from that followed in the aural type, owing to the essential difference between the signals used for marking out the beacon courses in the two sys-

[1] Applying the Radio Range to the Airways, by Kear and Jackson. B. S. Jour. Research.

tems. With the aural system, a pilot's position on either side of the course is indicated by the relative strength of two Morse characters. When on the course these characters blend into one long dash of constant signal strength. In order to obtain proper interlocking no portions of the two characters used can be transmitted simultaneously. With the visual system, a course is indicated by the equality of vibration amplitude of two reeds mechanically tuned to the two modulation frequencies used at the beacon. For maximum reed amplitude, it is desirable to provide continuous modulation at both modulation frequencies. The two distinguishing signals of the beacon are, therefore, always sent out simultaneously, resulting, under normal conditions, in a space pattern quite different from that obtained from an aural radio range. This was shown by Pratt in Figure 2, page 875, Proceedings of the Institute of Radio Engineers, May, 1929. Two, rather than four, beacon courses are obtained. To produce four beacon courses it is necessary to advance the time phase of one of the

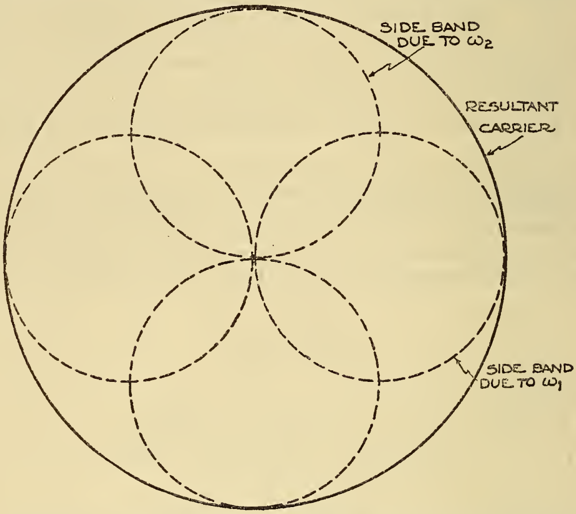


Figure 1.—*Space pattern radiated by double-modulation type radio range when antenna currents are 90° out of phase*

carrier frequencies of the system 90° beyond that of the other. The space pattern is then as indicated in Figure 4, page 877, of Pratt's paper, the four beacon courses being displaced by 90° from each other. For convenience in reference, this figure is reproduced here as Figure 1. The received polar pattern is shown in Figure 2. The trigonometric expression for the beacon space pattern is given in equation (1). This is equivalent to equation (5), page 876, of Pratt's paper. The corresponding expression for the polar pattern as received on the reeds is given in equation (2), which assumes square law detection.

$$e_p = K \left\{ \begin{aligned} & E_0 \left[\cos \omega t \sin \theta + \sin \omega t \cos \theta\right] \\ & \qquad - \frac{E_1}{2} \left[\sin\,(\omega - \omega_1)\,t - \sin\,(\omega + \omega_1)\,t\right] \sin \theta \\ & \qquad + \frac{E_2}{2} \left[\cos\,(\omega - \omega_2)\,t - \cos\,(\omega + \omega_2)\,t\right] \cos \theta \end{aligned} \right\} \tag{1}$$

where $e_p$ is the field intensity at any point $P$ in space as a polar function of the angle $\theta$, and, as is apparent, consists of a carrier and two sets of side bands; $\frac{E_1}{E_0}\times 100$ is the percentage modulation in amplifier branch 1 due to $\omega_1$; $\frac{E_2}{E_0}\times 100$ is the percentage modulation in amplifier branch 2 due to $\omega_2$. In this and all the following equations we will assume $E_1 = E_2 = E_0$ unless otherwise stated.

$$e_r = K\ K'\ E_0\{E_1 \sin \omega_1 t \sin^2 \theta + E_2 \sin \omega_2 t \cos^2 \theta\} \tag{2}$$

where $e_r$ is the received signal strength at any point $P$ as a polar function of the angle $\theta$. A course occurs whenever the two terms to the right of the equality sign of equation (2) are equal.

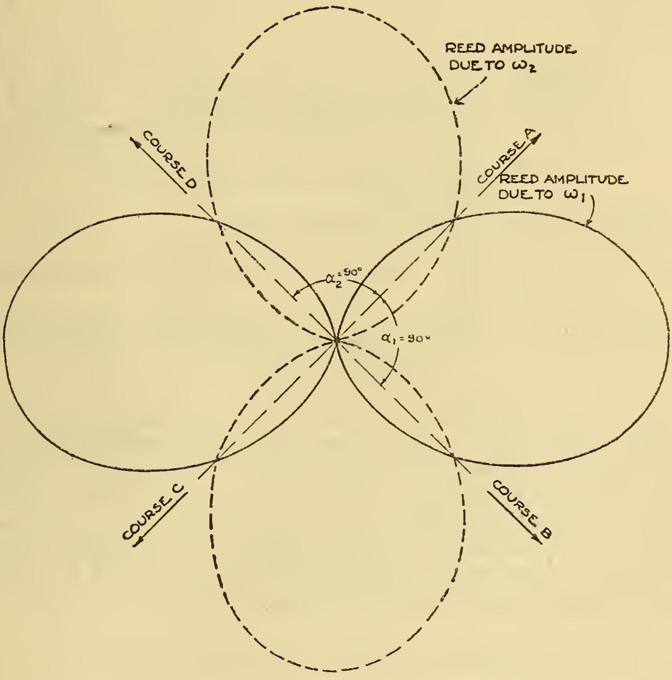


FIGURE 2.—*Received polar pattern corresponding to the space pattern of Figure 1*

From a study of equations (1) and (2), it is interesting to note that the useful signal due to $\omega_1$ is the result of the beating of the carrier of branch 1 (*not* the resultant carrier) with the side bands transmitted by branch 1. Similarly, the useful signal due to $\omega_2$ is the result of the beating of the carrier and side bands transmitted by branch 2. These relationships are true, however, only in the special case when the time phase between the two carriers of the system is equal to 90°.

A schematic diagram of the circuit arrangement used for obtaining four beacon courses at 90° with each other is shown in Figure 3. The means for producing a 90° time-phase displacement between the two carrier-frequency currents of the system consists simply of inserting a suitable capacitive reactance in the supply lead from the master oscillator to one amplifier branch and a suitable inductive reactance in the supply lead to the other amplifier branch. For convenience in phase adjustment the condenser is made variable, and a variable resistor is connected in series with each reactance.

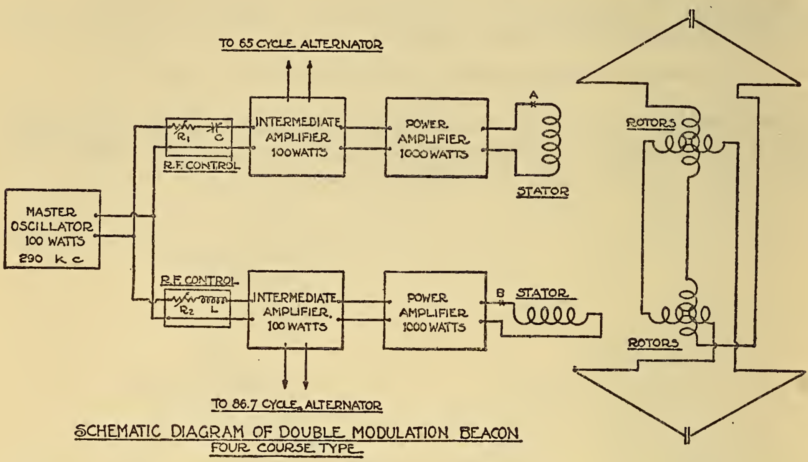


Figure 3.—*Schematic diagram of double-modulation radio range, four-course type*

## II. METHOD A, SHIFTING COURSES BY AMPLITUDE REDUCTION

One method of shifting the four beacon courses from their 90° space relationship is to reduce the percentage modulation of one amplifier branch, keeping the magnitude of the carrier unchanged. The space pattern radiated by the beacon is then as shown in Figure 4, and the received polar diagram as shown in Figure 5. A 30 per cent reduction in the percentage modulation of one amplifier branch is assumed. The trigonometric equations for the radiated space pattern and received polar pattern are then, respectively:

$$e_p = K\Big[E_0\{\cos \omega t \sin \theta + \sin \omega t \cos \theta\} - \frac{0.7E_1}{2}\{[\sin(\omega-\omega_1)t - \sin(\omega+\omega_1)t] \sin\theta\} + \frac{E_2}{2}\{[\cos(\omega-\omega_2)t - \cos(\omega+\omega_2)t] \cos\theta\}\Big] \tag{3}$$

$$e_r = K\,K' E_0\,[0.7E_1 \sin \omega_1 t \sin^2\theta + E_2 \sin \omega_2 t \cos^2\theta] \tag{4}$$

A course occurs whenever the two terms to the right of the equality sign of equation (4) are equal. It will be observed from Figure 5 that the two sets of 180° courses ($A$, $B$) and ($C$, $D$) are now displaced by $\alpha_1 = 80°$ and $\alpha_2 = 100°$ as compared with $\alpha_1 = \alpha_2 = 90°$ when the degree of modulation in the two amplifier branches is the same.

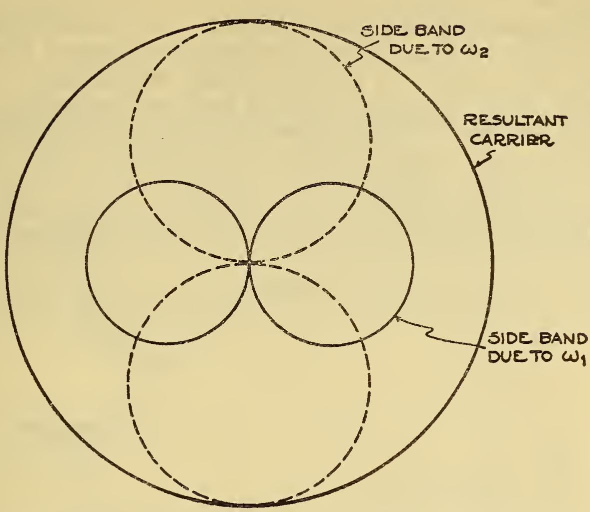


FIGURE 4.—*Space pattern when percentage modulation in one amplifier branch is reduced*

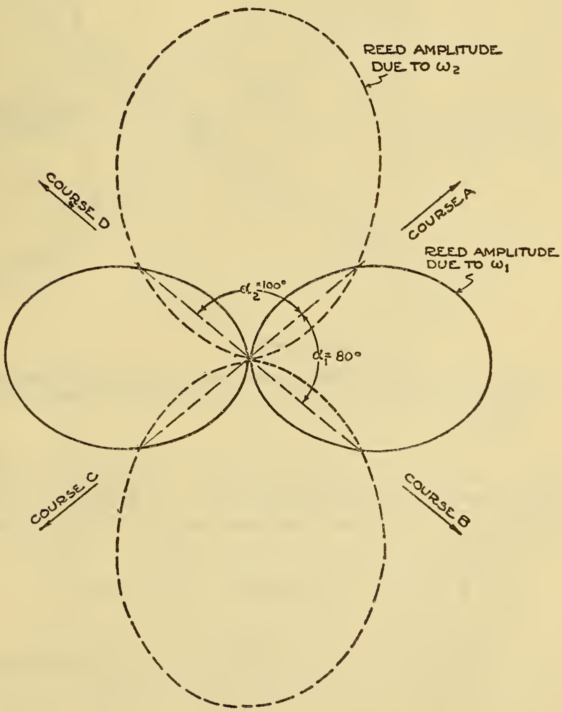


FIGURE 5.—*Received polar pattern corresponding to Figure 4*

The method of modulation adopted in the visual radio-range transmitter, consisting of plate excitation of the intermediate amplifiers from sources of a.c. supply of suitable frequencies, does not permit an easy adjustment of the percentage modulation. It is more convenient to reduce the magnitude of both carrier and side bands of one amplifier branch. This is accomplished by increasing the value of the resistor $R_1$ of the radio-frequency control (fig. 3) and readjusting $C$ to keep the phase of the voltage applied to the grid of the intermediate amplifier tube unchanged. Assuming a 30 per cent reduction in the amplitude of carrier and side bands of one amplifier branch, the

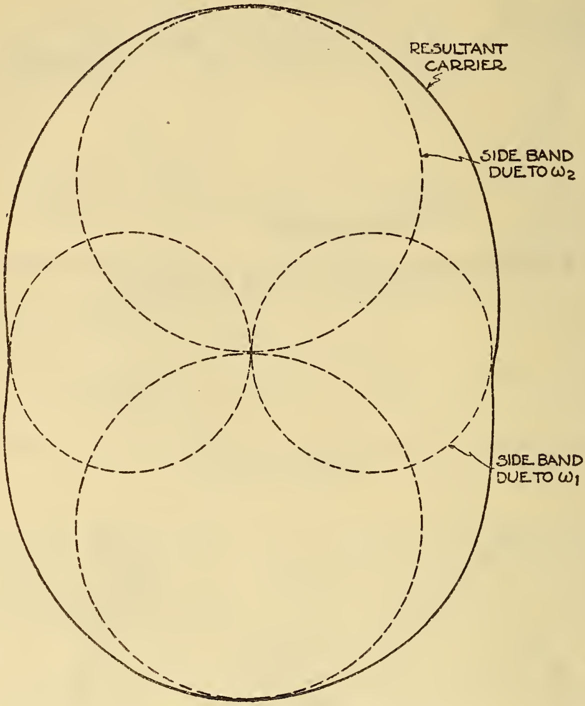


FIGURE 6.—*Space pattern when magnitude of the carrier and side bands transmitted by one amplifier branch is reduced*

radiated space pattern becomes that of Figure 6 while the polar diagram as received on the reeds is shown in Figure 7. The trigonometric expression for the space pattern is given in equation (5).

$$\begin{aligned} e_p = K\Big[ & E_0\{0.7 \cos \omega t \sin \theta + \sin \omega t \cos \theta\} \\ & -0.7\frac{E_1}{2}\{[\sin(\omega-\omega_1)t - \sin(\omega+\omega_1)t] \sin \theta\} \\ & +\frac{E_2}{2}\{[\cos(\omega-\omega_2)t - \cos(\omega+\omega_2)t] \cos \theta\}\Big] \end{aligned} \tag{5}$$

The corresponding expression for the received signal becomes

$$e_r = K\,K'E_0\,[0.49E_1 \sin \omega_1 t \sin^2 \theta + E_2 \sin \omega_2 t \cos^2 \theta] \tag{6}$$

A course will occur whenever the two reed deflections are of equal magnitude; that is, when

$$|0.7 \sin \theta| = |\cos \theta| \tag{7}$$

Equation (7) follows from the facts that the observed reed deflections are determined by the maximum instantaneous values of $E_1 \sin \omega_1 t$ and $E_2 \sin \omega_2 t$ and that the reeds are adjusted for equal

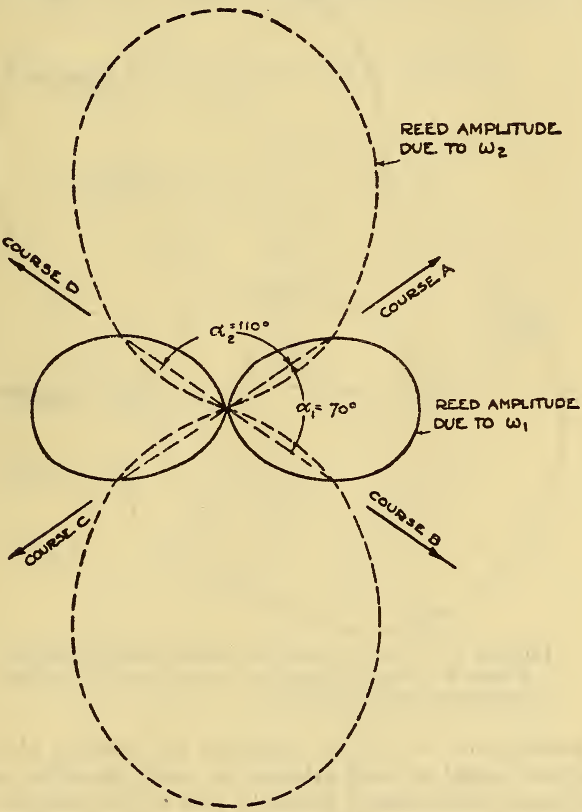


FIGURE 7.—*Received polar pattern corresponding to Figure 6*

sensitivity. Note that the courses have been shifted in greater amount than in Figure 5. The values for the angles $\alpha_1$ and $\alpha_2$ are now 70° and 110,° respectively. The relative reduction in strength of signal received when on course is, however, also greater.

88500°—30——7

Two factors determine the minimum value for $\alpha_1$ (or the maximum value for $\alpha_2$) that can be obtained: (*a*) The signal strength received when on any given course should not be reduced below 50 per cent of that received when the four courses are displaced by 90°, (*b*) a deviation of at least 20° on either side of the course should be possible without losing indications as to the direction back to the course. Keeping in mind these requirements, both methods described above yield as tolerable limits ($\alpha_1 = 60°$ $\alpha_2 = 120°$). Using these methods then it becomes possible to fit two of the beacon courses to any two airways separated by an angle in the range 60° to 120°.

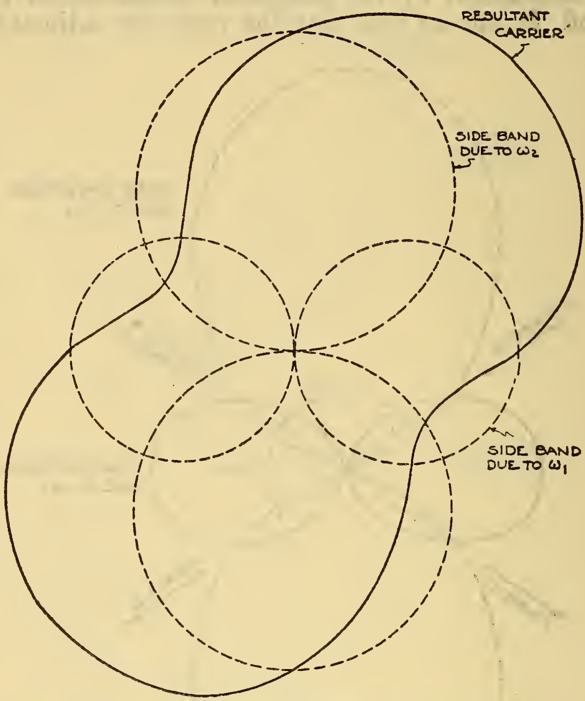


FIGURE 8.—*Space pattern for same conditions as in Figure 6 except that antenna currents are 45° out of time phase instead of 90°*

Before passing on to further methods for making the other two beacon courses useful, it is of interest to study the effect of changing the time-phase displacement between the two modulated waves of the transmitting system. As noted above, the difference in time phase is normally adjusted to 90°. Suppose that the time-phase difference is made 45° (by adjusting $L$ and $C$, fig. 3). For the condition that the magnitude of carrier and side bands transmitted by one amplifier branch is reduced to 70 per cent that of the other amplifier branch, the beacon space pattern becomes that shown in Figure 8 and the received polar pattern as shown in Figure 9. The expression for the radiated space pattern under these conditions is given in equation (8) and for the received polar pattern in equation (9).

$$e_p = K\left\{\begin{aligned} &E_0\left[0.7 \sin\left(\omega t-\frac{\pi}{4}\right)\sin\theta+\sin\omega t\cos\theta\right]\\ &+0.7\,\frac{E_1}{2}\left[\cos\left(\omega t-\frac{\pi}{4}-\omega_1 t\right)-\cos\left(\omega t-\frac{\pi}{4}+\omega_1 t\right)\right]\sin\theta\\ &+\frac{E_2}{2}[\cos(\omega-\omega_2)t-\cos(\omega+\omega_2)t]\cos\theta \end{aligned}\right\} \qquad (8)$$

$$\begin{aligned} e_r = KK'E_0\Bigg[&E_1\sin\omega_1 t\left\{0.7\times\frac{1}{\sqrt{2}}\sin\theta\cos\theta+0.49\sin^2\theta\right\}\\ &+E_2\sin\omega_2 t\left\{0.7\times\frac{1}{\sqrt{2}}\sin\theta\cos\theta+\cos^2\theta\right\}\Bigg] \end{aligned} \qquad (9)$$

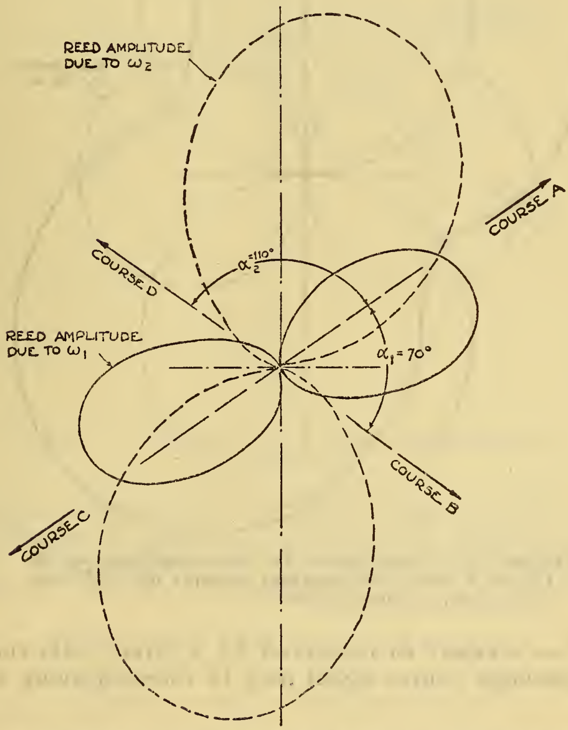


FIGURE 9.—*Received polar pattern corresponding to Figure 8*

Note that since the two carrier frequency currents of the system are no longer displaced in time phase by 90°, the relationship observed in connection with equations (1) and (2) no longer holds.

If the time phase displacement is made 135°, the resultant space pattern is as shown in Figure 10 and the received polar pattern as shown in Figure 11. The trigonometric equations corresponding to these patterns are similar to equations (8) and (9) and will not be given here. Note that a change in the time phase displacement does not result in a shifting of the beacon courses. An exact adjustment of time-phase displacement to 90° is therefore not necessary. By varying this displacement, however, it is possible to control at will the relative signal strength on the two sets of 180° courses. In this

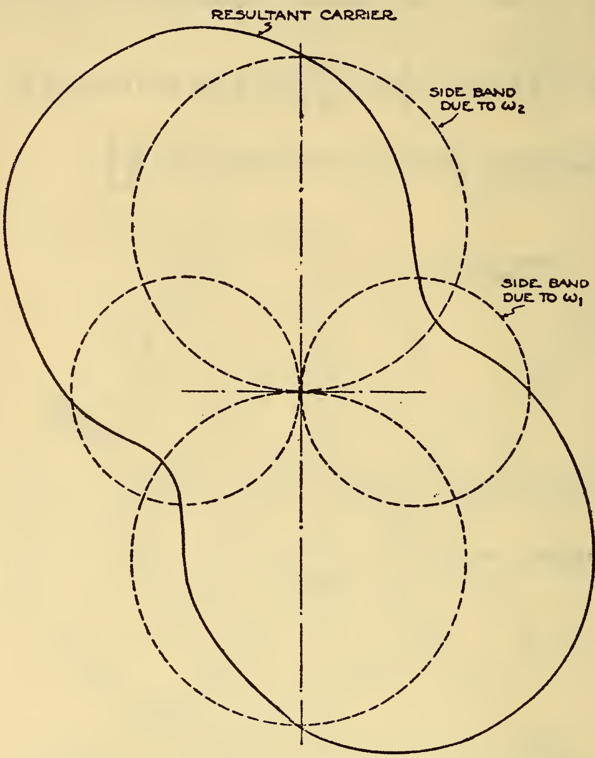


FIGURE 10.—*Space pattern for same conditions as in Figure 6 except that antenna currents are 135° out of time phase instead of 90°*

way, if the two airways to be served by a given radio range differ in length, a stronger course signal may be directed along the longer airway.

## III. METHOD B, USE OF AUXILIARY VERTICAL ANTENNA FOR SHIFTING COURSES

A further method for shifting the courses from their 90° space relationship consists of supplying a circular radiation of the carrier and side bands transmitted by one amplifier branch, in addition to the figure-of-eight radiation due to the loop antenna fed by that amplifier branch. A vertical antenna, running the length of the beacon tower and inductively coupled to the output circuit of one of the two amplifying branches of the transmitting system, is employed for obtaining this additional radiation. Assuming a ratio of amplitude of circular radiation to maximum amplitude of figure-of-

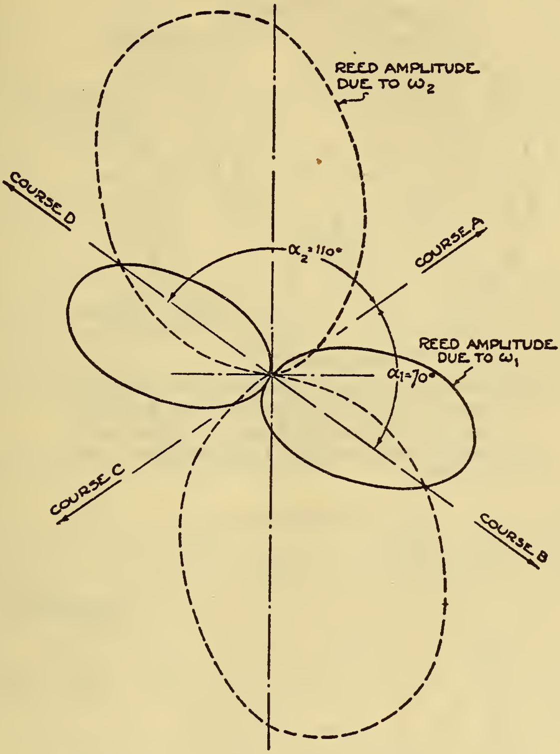


FIGURE 11.—*Received polar pattern corresponding to Figure 10*

eight radiation equal to 0.28, the radiated space pattern becomes as shown in Figure 12, and the received polar diagram as shown in Figure 13. The expression for the radiated space pattern is given in equation (10) and for the received pattern in equation (11).

$$e_p = K\left\{\begin{array}{l} E_0\,[\cos\,\omega t\,\sin\,\theta + \sin\,\omega t\;(0.28 + \cos\,\theta)] \\ -\dfrac{E_1}{2}[\sin\;(\omega - \omega_1)\,t - \sin\;(\omega + \omega_1)\,t]\,\sin\,\theta \\ +\dfrac{E_2}{2}[\cos\;(\omega - \omega_2)\,t - \cos\;(\omega + \omega_2)\,t]\,[0.28 + \cos\,\theta] \end{array}\right\} \tag{10}$$

$$e_r = KK'E_0\{E_1\,\sin\,\omega_1 t\,\sin^2\,\theta + E_2\,\sin\,\omega_2 t\,[0.28 + \cos\,\theta]^2\} \tag{11}$$

A beacon course will occur when equation (12) is satisfied.

$$|\sin\,\theta| = |\,(028 + \cos\,\theta)\,| \tag{12}$$

In equation (10) the current in the vertical antenna is taken as in time phase with the current in loop antenna (2). This is the condition of maximum course shift for a given vertical antenna current. If these two currents were in time quadrature, no course shift would

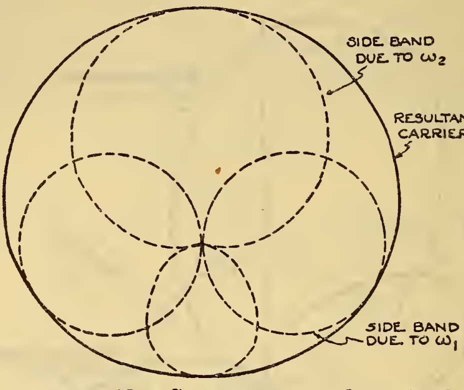


FIGURE 12.—*Space pattern when circular radiation of the modulated wave transmitted by one amplifier branch is added to the normal figure-of-eight radiation due to that branch*

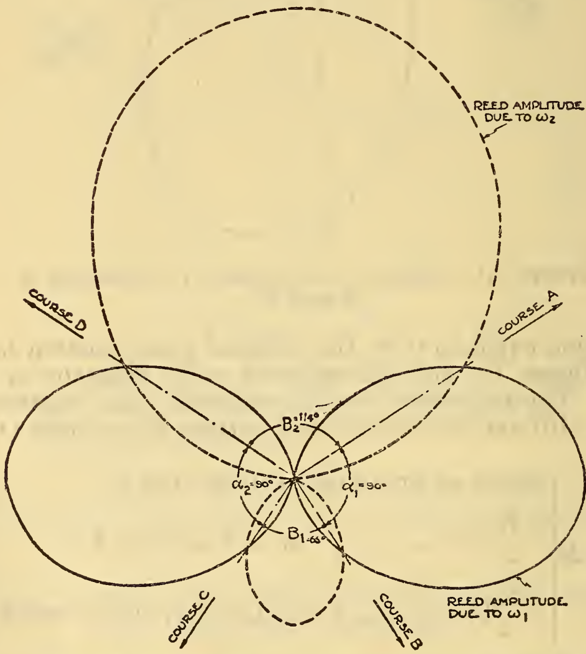


FIGURE 13.—*Received polar pattern corresponding to Figure 12*

occur. It is, therefore, important to insure that these currents are in phase; (*a*) by keeping the vertical antenna and loop antenna (2) accurately in tune, and (*b*) by connecting the primary of the coupling transformer to the vertical antenna in the output circuit of the same amplifier stage in which the primary winding of the coupling arrangement to loop antenna (2) is connected (that is, at point $B$, fig. 3).

Comparing the received polar diagram of Figure 13 with that for the beacon normally adjusted (see fig. 2), it will be observed that the addition of a circular component to the normal radiation characteristic of the modulated wave due to $\omega_2$ results in decreasing the angle $\beta_1$ between courses $B$ and $C$ and increasing the angle $\beta_2$ between courses $D$ and $A$. The angles $\alpha_1$ between courses $A$ and $B$ and $\alpha_2$ between courses $C$ and $D$ remain equal to 90°. Applying the same criterions as were used in the method of course-shifting by amplitude reduction (namely, that the signal strength on course should not be reduced below 50 per cent of normal, and that a deviation of 20° on either side of a course be possible), the minimum tolerable value for $\beta_1$ is 60° and the maximum value for $\beta_2$ is 120°.

A comparison of methods A and B for varying the angle between two beacon courses is of interest. Using method A, two airways separated by any angle in the range 60° to 120° may be served. This is also true using method B. In method A, however, each of the two remaining beacon courses is displaced by 180° from one of the two courses used, while in method B each of the two remaining courses is displaced by 90° from one of the two courses employed.

## IV. METHOD C, COMBINATION OF METHODS A AND B

Methods A and B may be combined, yielding an arrangement for serving three airways simultaneously, provided the angles between these airways are within the limits tabulated below. For convenience in tabulation, the three airways are called A, B, and C′, respectively. Any two of these (say A and B) may be from 60° to 90° apart. The third airway (C′) may then be disposed from either A or B by any angle within the range given in the table. The two criterions noted above under methods A and B are fulfilled in this table. As shown, the practicable range of angles between C′ and either A or B depends upon the angle between A and B. A greater variation can, of course, be obtained if it is permissible to reduce the signal strength received when on course. The third column of Table 1 shows the possible range if the minimum permissible signal strength when on any given course is taken as 33 per cent of normal.

TABLE 1

| Angle between $A$ and $B$ | Range of permissible angles between $C'$ and either $A$ or $B$ if $\frac{E_r'}{E_r}=0.5$ | Range of permissible angles between $C'$ and either $A$ or $B$ if $\frac{E_r'}{E_r}=0.33$ |
|---|---|---|
| 60° | 120°, 180° | 90°–210°. |
| 65° | 105°–125°, 170°–190° | 80°–215°. |
| 70° | 90°–130°, 160°–200° | 70°–220°. |
| 75° | 82.5°–127.5°, 157.5°–202.5° | 65°–220°. |
| 80° | 75°–125°, 155°–205° | 60°–220°. |
| 85° | 67.5°–122.5°, 152.5°–207.5° | 55°–135°, 140°–220°. |
| 90° | 60°–120°, 150°–210° | 50°–130°, 140°–220°. |

The results indicated in Table 1 are obtained by reducing the amplitude of carrier and side bands transmitted by one amplifier branch to obtain any angle from 60° to 90° between courses A and B, and then introducing a circular radiation of the carrier and side bands transmitted by the other amplifier branch in addition to the normal figure-of-eight radiation for that branch. The space pattern as radiated from the beacon then becomes as shown in Figure 14 and the received polar diagram in Figure 15. The trigonometric equation for the radiated space pattern is given by equation (13) and for the received pattern by equation (14).

$$e_p = K\left\{\begin{array}{l} E_0\,[C_1 \cos \omega t \sin \theta + \sin \omega t\ (K_2 + \cos \theta)] \\ \quad - C_1 \dfrac{E_1}{2}\,[\sin\,(\omega - \omega_1)t - \sin\,(\omega + \omega_1)t] \sin \theta \\ \quad + \dfrac{E_2}{2}\,[\cos\,(\omega - \omega_2)t - \cos\,(\omega + \omega_2)t]\,[K_2 + \cos \theta] \end{array}\right\} \tag{13}$$

where

$$K_2 = \frac{\text{amplitude of circular radiation due to } \omega_2}{\text{maximum amplitude of figure-of-eight radiation due to } \omega_2}$$

$C_1$ = reduction factor for carrier and side bands due to $\omega_1$

$$e_r = KK'\,E_0\,\{C_1^2 E_1 \sin \omega_1 t \sin^2 \theta + E_2 \sin \omega_2 t\,[K_2 + \cos \theta]^2\} \tag{14}$$

In Figures 14 and 15, $C_1 = 0.7$ and $K_2 = 0.22$.

Referring to equation (14), a course will occur whenever

$$|K_2 + \cos \theta| = |C_1 \sin \theta| \tag{15}$$

This equation will have four solutions, one for each quadrant. In the first quadrant

$$K_2 + \cos \theta_1 = C_1 \sin \theta_1 \qquad \theta_1 = \text{angle of course } A \tag{16}$$

In the second quadrant

$$K_2 - \cos \theta_2 = C_1 \sin \theta_2 \qquad \theta_2 = \text{angle of course } B \tag{17}$$

In the third quadrant

$$K_2 - \cos \theta_3 = -C_1 \sin \theta_3 \qquad \theta_3 = \text{angle of course } C \tag{18}$$

In the fourth quadrant

$$K_2 + \cos \theta_4 = -C_1 \sin \theta_4 \qquad \theta_4 = \text{angle of course } D \tag{19}$$

In equations (16), (17), (18), and (19) the factors $\sin \theta_1$, $\cos \theta_1$, etc., are constants of positive sign.

Subtracting (17) from (16) and solving, we obtain

$$\theta_2 - \theta_1 = 2\alpha \qquad \text{where } \alpha = \tan^{-1} C_1 \tag{20}$$

Similarly, subtracting (19) from (18) and solving, we obtain

$$\theta_4 - \theta_3 = 2\alpha \tag{21}$$

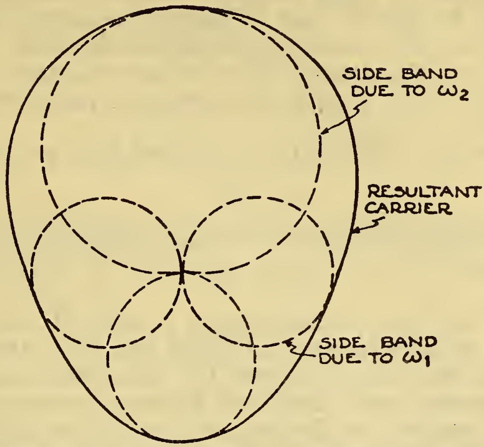


FIGURE 14.—*Space pattern when methods A and B for course shifting are combined*

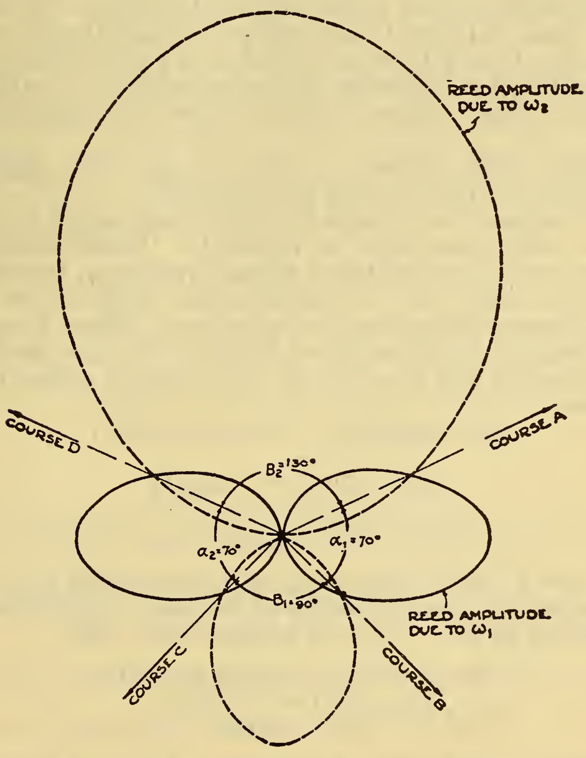


FIGURE 15.—*Received polar pattern corresponding to Figure 14*

We note, then, that the angle between courses A and B, and likewise between courses C and D, are dependent upon $C_1$, the reduction factor of carrier and side bands in one amplifier branch. These angles are independent of the amount of circular radiation of carrier and side bands added to the normal figure-of-eight radiation of the other amplifier branch.

Again, subtracting (18) from (17) and solving, we obtain

$$\theta_3 = -\theta_2 \tag{22}$$

Similarly, subtracting (16) from (19) and solving, we obtain

$$\theta_4 = -\theta_1 \tag{23}$$

Equations (22) and (23) indicate that courses B and C, and also courses D and A, are symmetrical about the 0° to 180° axis. This may be seen by reference to Figure 15. The angle between courses B and C is decreased, and the angle between courses D and A increased in like amount, as the ratio of maximum amplitude of circular to figure-of-eight radiation due to the second amplifier branch is increased. If the phase of the circular radiation is reversed with respect to the phase of the figure-of-eight radiation, an increase in this ratio will result in an increase in the angle between courses B and C and a decrease in the angle between D and A. In either case, for a given fixed value of $C_1$, the angles between the courses A and B and between C and D remain fixed at a value 2 $\tan^{-1} C_1$.

The procedure of determining the proper values for $C_1$ and for $K_2$ in order to serve three courses at given angles with each other, is as follows:

(*a*) Suppose that courses A and B are 75° apart. Place $2 \tan^{-1} C_1 = 75°$. Then $C_1 = \tan 37.5° = 0.767$.

(*b*) Now, suppose that course C′ is 90° from course B. For convenience in fixing ideas, refer to Figure 15. Under normal adjustments, that is, with no additional vertical radiation, course C would be 105° from course B. To decrease this angle to 90°, circular radiation of proper phase must be introduced. Since courses B and C are symmetrically disposed about the 0° to 180° axis, and the angle between courses B and C equals 90°, the angle of course C must be 225°. Place

$$|K_2 + \cos(225°)| = |0.767 \sin(225°)|$$

then

$$|K_2 - 0.707| = |-0.542|$$

and

$$K_2 = 0.165$$

(*c*) If course C′ were to be, say, 120° from course B, the vertical radiation to be introduced must be of the same magnitude as in (*b*) but of opposite phase. Thus, $\theta_3$ becomes 240° and

$$|K_2 + \cos 240°| = |0.767 \sin 240°|$$
$$|K_2 - 0.5| = |-0.665|$$
$$K_2 = -0.165$$

(*d*) Suppose that an angle of 165° between courses B and C′ were desired. Course D is normally 180° from B and is therefore the course to be used, with the proper amount of circular radiation introduced to obtain the desired angular shift. Courses D and A are symmetrically displaced about the 0° to 180° axis and course A is 75° from B. The angle of course D is therefore

$$\theta_4 = 300°$$

and

$$|K_2 + \cos 300°| = |0.767 \sin 300°|$$
$$|K_2 + 0.50| = |-0.665|$$
$$K_2 = 0.165$$

Note that for this value of $K_2$, course C is 90° from B.

(*e*) Again, suppose that an angle of 195° is desired between courses B and C′. Course D will again be used but circular radiation of negative sign is necessary.

$$\theta_4 = 315°$$
$$|K_2 + \cos 315°| = |0.767 \sin 315°|$$
$$|K_2 + 0.707| = |-0.542|$$
$$K_2 = -0.165$$

Note that for this value of $K_2$, course C is 120° from B.

## V. METHOD D, EXTENSION OF METHOD B

It is frequently desirable to shift two of the four-beacon courses from their 180° relationship (viz, *B* and *D*) without disturbing the 180° relationship between the other two beacon courses (A and C). This can be accomplished by the introduction of circular radiation in equal amounts of the carrier and side bands transmitted by both amplifier branches in addition to their normal figure-of-eight radiation. Figure 16 shows the beacon space pattern for a case of this type, and Figure 17 shows the corresponding received polar pattern. The mathematical expressions corresponding to these patterns are given in equations (24) and (25), respectively.

$$e_p = K\left\{\begin{array}{l} E_0 [\cos \omega t\ (K_1 + \sin \theta) + \sin \omega t\ (K_2 + \cos \theta)] \\ \quad -\frac{E_1}{2} [\sin (\omega - \omega_1)\ t - \sin (\omega + \omega_1)\ t]\ [K_1 + \cos \theta] \\ \quad +\frac{E_2}{2} [\cos (\omega - \omega_2)\ t - \cos (\omega + \omega_2)\ t]\ [K_2 + \cos \theta] \end{array}\right\} \tag{24}$$

where

$$K_1 = \frac{\text{amplitude of circular radiation due to } \omega_1}{\text{maximum amplitude of figure-of-eight radiation due to } \omega_1}$$

$$K_2 = \frac{\text{amplitude of circular radiation due to } \omega_2}{\text{maximum amplitude of figure-of-eight radiation due to } \omega_2}$$

$$e_r = KK'E_0\{E_1 \sin \omega_1 t\ [K_1 + \sin \theta]^2 + E_2 \sin \omega_2 t\ [K_2 + \cos \theta]^2\} \tag{25}$$

A course will occur when equation (26) is satisfied.

$$|K_1 + \sin \theta| = |K_2 + \cos \theta| \tag{26}$$

In Figures 16 and 17, $K_1 = K_2 = 0.2$

Applying method D, the minimum practicable angle that can be obtained between courses B and D is 145°. This angle can be made smaller, however, if the weakest course (C) is not to be used. For example, suppose that a radio range installed at Richmond, Va., is

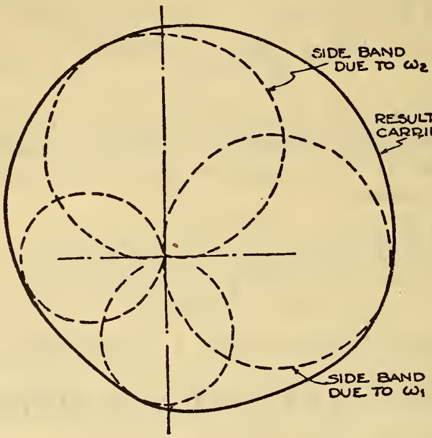


FIGURE 16.—*Space pattern when circular radiation in equal amounts of the modulated waves transmitted by both amplifier branches is added to their normal figure-of-eight radiation*

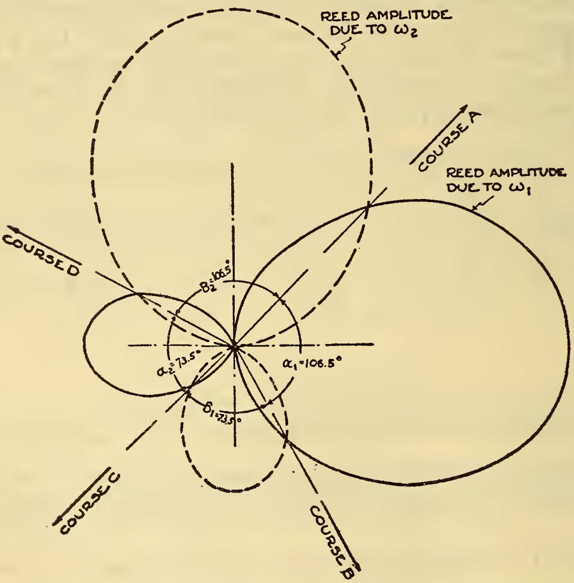


FIGURE 17.—*Received polar pattern corresponding to Figure 16*

to serve the routes to Quantico, Va., Norfolk, Va., and Greensboro, N. C. The bearings of these cities from Richmond are, respectively, 1°, 126°, and 237°. These are plotted in full lines in Figure 18, together with the proper beacon space pattern for serving the routes to these cities. The method of arriving at this pattern is of interest. Allowing an error of 0.5°, the courses to Norfolk, Va., and Greensboro, N. C., are equally displaced from the course to Quantico, Va. The bearings of two of the three courses may, therefore, be substituted successively in equation (26), and the values for $K_1$ and $K_2$ determined. Since the course directed on Quantico, which has a bearing of 1°, corresponds, obviously, to course A (fig. 17), having

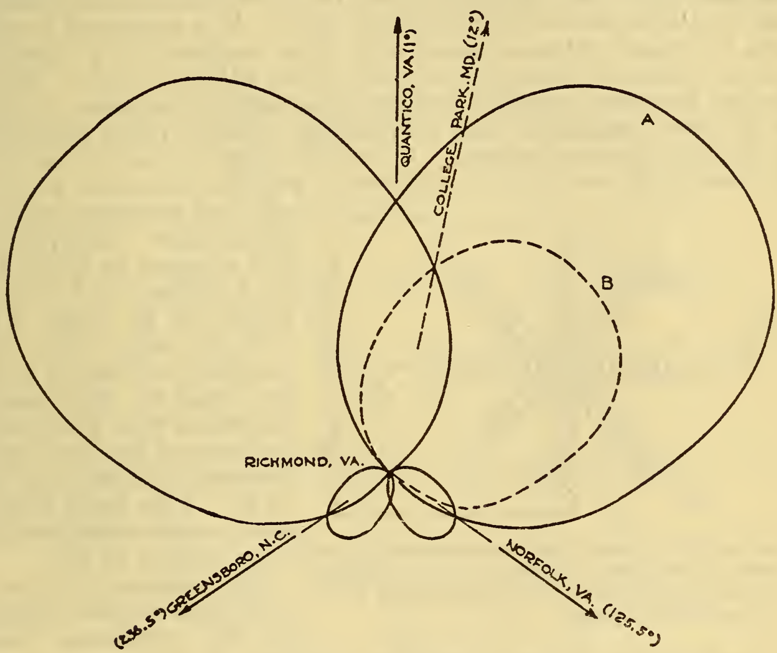


FIGURE 18.—*Application of the method of course-shifting illustrated by Figures 16 and 17 to actual airway routes*

a bearing of 45°, a correction factor of (+44°) to the bearings of the two courses to be substituted in equation (26) must be applied. Thus

$$|K_1+\sin(1°+44°)|=|K_2+\cos(1°+44°)|$$

$$|K_1+\sin(125.5°+44°)|=|K_2+\cos(125.5°+44°)|$$

Solving, these two equations simultaneously, we have

$$K_1=0.40$$

$$K_2=0.40$$

A verification that these values are correct may be had by determining

whether $|0.4+\sin(236.5°+44°)|=|0.4+\cos(236.5°+44°)|$

Solving $|0.582|=|-0.582|$, which is correct.

Some difficulty arises in the application of this method due to the fact that no coupling should exist between amplifier branches of the beacon transmitter. Referring to Figure 3, introducing circular radiation of the modulated waves transmitted by both amplifier branches of the system involves coupling the vertical antenna to suitable inductances inserted at $A$ and at $B$. This at once introduces coupling between amplifiers and tends to destroy the beacon pattern. To overcome this difficulty the arrangement indicated in Figure 19 must be employed. The inductances, $S_1$ and $S_2$, are inserted at $A$ and at $B$ as previously, but $S_1$ and $S_2$ are crossed at 90° with each other. The secondary coils, $R_1$ and $R_2$, are mounted concentrically with $S_1$ and $S_2$ and are also at 90° with each other. $R_1$ is connected in series with the tuned antenna circuit while $R_2$ is connected in a tuned circuit of constants identical with those of the antenna circuit. $R_1$ and $R_2$ may be made rotatable, making the coupling arrangement similar to the 4-coil goniometer used for coupling the radio range transmitter to the loop antenna system. As was shown in the case of the 4-coil goniometer, no coupling exists between amplifier branches, provided the loop antennas are in accurate tune. (See pp. 910–912 of paper on Radio Aids to Aviation, by J. H. Dellinger and H. Pratt, Proc. I. R. E.; July, 1928.) The coupling between $S_1$ and $S_2$ by virtue of their mutual induction with $R_1$ is exactly neutralized by the coupling between $S_1$ and $S_2$ by way of $R_2$. This holds true for every angular position of the rotor system $R_1$ $R_2$.

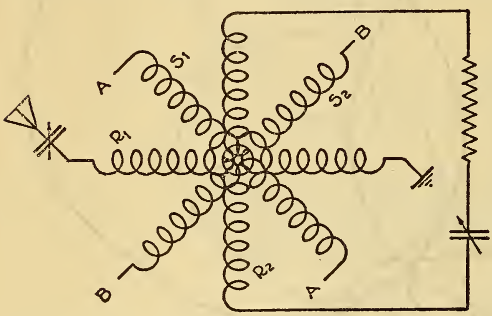

Figure 19.—*Neutralizing arrangement to prevent intercoupling between the two amplifier branches even though these are coupled to the same vertical antenna*

The position of $R_1$ to obtain equal circular radiation of the modulated waves transmitted by the two amplifier branches is, of course, at 45° with $S_1$ and $S_2$. Changing the angular setting of $R_1$ results in a change in the relative amount of circular radiation for the two modulated waves. This provides an added feature of flexibility, since it makes it possible to shift the two sets of 180° courses (A, C, and B, D) from their 180° relationship in varying amounts. This may best be illustrated by the results collected in Table 2. The received polar diagrams shown in Figures 13 and 17 correspond, respectively, to the 90° and 45° rotor settings in Table 2.

Table 2

| Rotor setting | $K_2$ | $K_1$ | Angle of course A | Angle of course B | Angle of course C | Angle of course D | Angle between courses C and A | Angle between courses D and B |
|---|---|---|---|---|---|---|---|---|
| ° | | | ° | ° | ° | ° | ° | ° |
| 0 | 0 | 0.283 | 33.5 | 146.5 | 236.5 | 303.5 | 203 | 157 |
| 14 | .068 | .274 | 36.6 | 149 | 233.4 | 301.0 | 196.8 | 152 |
| 18.5 | .090 | .268 | 37.5 | 149.8 | 232.5 | 300.2 | 195 | 150.4 |
| 26.5 | .126 | .253 | 40 | 150.5 | 230 | 299.5 | 190 | 149 |
| 45 | .200 | .200 | 45 | 151.4 | 225 | 298.6 | 180 | 147.2 |
| 63.5 | .253 | .126 | 50 | 150.5 | 220 | 299.5 | 170 | 149 |
| 71.5 | .268 | .090 | 52.5 | 149.8 | 217.5 | 300.2 | 165 | 150.4 |
| 76 | .274 | .068 | 53.4 | 149 | 216.6 | 301 | 163.2 | 152 |
| 90 | .283 | 0 | 56.5 | 146.5 | 213.5 | 303.5 | 157 | 157 |

## VI. METHOD E, COMBINATION OF METHODS D AND A

A typical example will best explain the combination of methods D and A to give a flexible procedure for shifting the beacon courses so that they may coincide with the airways radiating from a given airport. Considering the bureau's experimental station at College Park, Md., suppose that it is desired to direct courses on Hatboro, Pa., Norfolk, Va., Quantico, Va., and Bellefonte, Pa. The bearings of

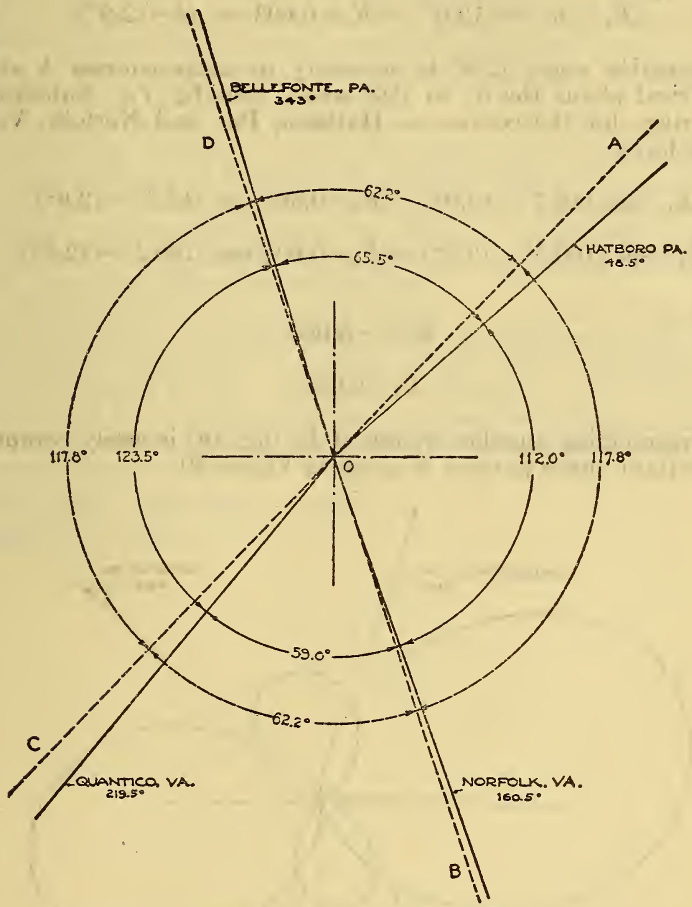


FIGURE 20.—*First approximation toward serving the four airway routes shown*

these cities from College Park are, respectively, Hatboro, Pa., 48.5°; Norfolk, Va., 160.5°; Quantico, Va., 219.5°; and Bellefonte, Pa., 343°. These courses are plotted in Figure 20, the angles between these courses being as indicated. The four courses can obviously be approximated by the method of amplitude reduction (method A) for shifting courses. Thus, draw line *AC* through the origin and equally displaced from the courses to Hatboro, Pa., and Quantico, Va. Also draw line *BD* through the origin and equally displaced from the courses to Norfolk, Va., and Bellefonte, Pa. The angle between these two lines is 62.2°. Placing $2 \tan^{-1} C_2 = 62.2°$ where $C_2$ is the reduction factor for the carrier and side band in amplifier branch 2, we obtain $C_2 = 0.603$. It is now necessary to introduce sufficient circular radiation of both modulated waves to make courses A, B, C, and D coincide with the desired routes. The proper values for $K_1$ and $K_2$ may be determined by substituting the bearings of two of the desired courses in equation (27)

$$|K_1 + \sin(\theta - 12.9°)| = |K_2 + 0.603 \cos(\theta - 12.9°)| \tag{27}$$

The correction angle 12.9° is necessary to make courses A and D symmetrical about the 0° to 180° axis. (See fig. 7.) Substituting the bearings for the courses to Hatboro, Pa., and Norfolk, Va., in (27), we have

$$|K_1 + \sin(48.5° - 12.9°)| = |K_2 + 0.603 \cos(48.5° - 12.9°)|$$

$$|K_1 + \sin(160.5° - 12.9°)| = |K_2 + 0.603 \cos(160.5° - 12.9°)|$$

Solving

$$K_1 = -0.060$$

$$K_2 = 0.033$$

The corresponding angular setting of $R_1$ (fig. 19) is easily computed. The resultant space pattern is given in Figure 21.

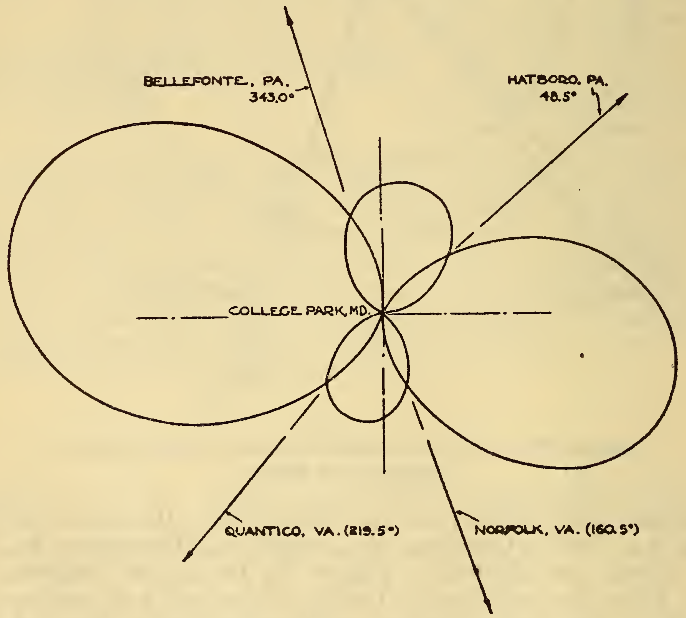


FIGURE 21.—*Final polar pattern for serving the four routes shown simultaneously*

## VII. METHOD F, SHIFTING THE COURSES BY CHANGES AT THE RECEIVING STATION

The methods for shifting the beacon courses as described above all require certain adjustments at the beacon transmitting station. Another possible method utilizes an adjustment of the receiving equipment aboard the airplane. This is accomplished by shunting a suitable resistance across the coil actuating one of the two reeds comprising the course indicator, thereby reducing the sensitivity of that reed. The course, as determined by equality of reed deflections, is therefore shifted from the true equisignal zone of the beacon in the direction of the shunted reed. The same effect is thus accomplished as by shifting the courses at the beacon.

This method may be used alone; that is, with the beacon normally adjusted for four courses at 90° to each other, or it may be used in conjunction with any of the methods outlined above. In the former case, a course shift of 15° on either side of the four beacon courses can be obtained. In the latter case, the degree of course shift possible (at the same time fulfilling the criteria set up under method A), depends upon the polar pattern employed.

An example of the latter case may be seen by reference to Figure 18. A course directed on College Park, Md., in addition to the three courses already indicated, is desired. The pilot may obtain this course in preference to the one directed on Quantico, Va., by shunting the proper reed by a suitable resistance, thereby obtaining the dotted line pattern B on that reed in place of the full line pattern A obtained with the reed unshunted.

WASHINGTON, September 11, 1929.

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