ON THE DETERMINATION OF THE MEAN HORIZONTAL INTENSITY OF INCANDESCENT LAMPS BY THE ROTATING LAMP METHOD.

By Edward P. Hyde and F. E. Cady.

INTRODUCTION.

There are several methods in use at the present time for the determination of the so-called mean horizontal intensity of incandescent lamps, i. e. the mean intensity in the plane perpendicular to the axis of the lamp and passing through its center. The oldest of these methods, and one which is equally applicable to other light sources, consists in measuring the intensity at equal angular intervals in the horizontal plane, and either taking the mean of the observed values, or plotting the observations and determining the mean radius vector of the curve drawn through the plotted points. A second method particularly adapted to the incandescent lamp because of the constancy of its curve of mean horizontal intensity was developed from the first method as an abbreviation of it. According to this second method photometric measurements are made in a single fixed direction, and the mean horizontal intensity is computed from mean horizontal reduction factors previously determined for the different types of lamps. Dyke has recently shown,[1] however, that even for lamps of a single type of filament individual differences of such magnitude occur as to produce serious errors in the computed values of mean horizontal intensity. This fact in conjunction with the inconvenience of the necessary orientation of the lamps on the photometer bar in making the measure-

[1] Phil. Mag., 9, p. 136; 1905.

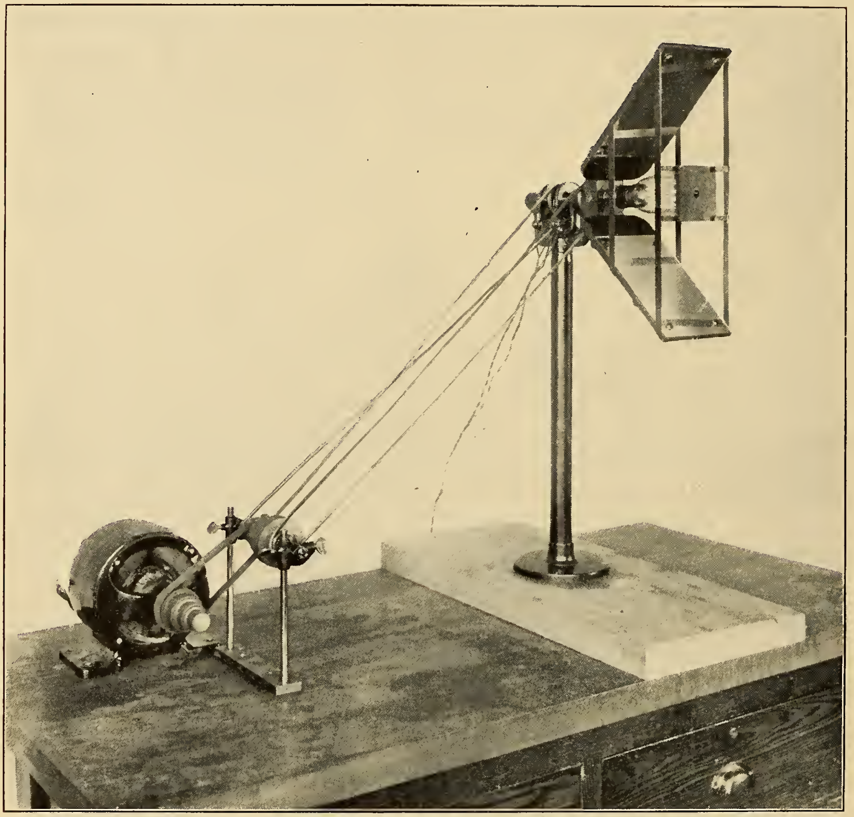

Fig. 1.—*Rotating Mirror System.*

ments has rendered the method of little practical value, and consequently its use at present is quite restricted.

In a modified form of this method which is used in Germany, the direct light in a definite direction from the lamp is supplemented by light reflected from two mirrors placed behind the lamp and inclined at an angle of 120°, so that an illumination is produced on the photometer screen, which is approximately proportional to the mean of the intensities in the three directions. Liebenthal has shown[2] the possible errors incident to this method, for the types of lamps studied, to be as large as 5 or 6 per cent when the lamps are definitely oriented, and 13 per cent when the lamps are set at random.

A third method, and one which is in almost universal use in this country in all practical determinations of the mean horizontal intensity of incandescent lamps, consists in spinning the lamp about its axis at a uniform speed of 180 revolutions per minute. The suggestion of spinning the lamp is said to have been made first by Prof. C. R. Cross at the National Electrical Congress held in Philadelphia, in 1884, but the writers have been unable so far to find a record of such a suggestion. Thirteen years later the Committee on Standardization of the American Institute of Electrical Engineers recommended[3] this method, suggesting a speed of 120 revolutions per minute. In recent years a speed of 180 revolutions per minute has been used quite universally, but an attempt to trace the authority for this modification has not as yet met with success.

A fourth method devised by Liebenthal[4] and used at the Physikalisch-Technische Reichsanstalt consists in rotating a pair of mirrors about the lamp held in a stationary position. The lamp is mounted with its axis horizontal and coincident with the photometric axis, the tip of the lamp being turned toward the photometer screen. The two mirrors, inclined at approximately 90° to each other, reflect to the photometer screen light emitted from the lamp in a direction perpendicular to its axis. The direct rays from the tip of the lamp are prevented from reaching the photometer by a small screen. The mirrors rotate about the axis of the lamp and reflect to the screen a quantity of light proportional to the mean

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[2] Zs. für Instrumentenkunde, **19**, p. 193; 1899.

[3] Trans. Amer. Inst. of Elec. Eng., **14**, p. 90; 1897.

[4] Loc. cit.

horizontal intensity of the source. This method necessitates, of course, the determination of the reflection coefficients of the mirrors, which is accomplished by means of a standard incandescent lamp, the mean horizontal intensity of which is known from a previous determination by the first method mentioned above.

Of the above four methods by far the most convenient in commercial testing is that of rotating the lamp, because but one measurement is required, and there is no question about the orientation of the lamp. Moreover, by simple mechanical devices a reversal of the direction of rotation may be obtained so that lamps fitted with screw bases can automatically be screwed into and out of the socket, thus materially lessening the time required for measurement. Although the method of rotating the lamp is the most convenient it has been open to serious objection on account of possible errors which have never been investigated, so that while the method is in quite general use in this country there are many who are in doubt in regard to its accuracy, and its adoption abroad has been restricted.

The two possible sources of error of this method are (1) the possible distortion of the filament, and (2) the effect of the flicker, which is perceptible in nearly all types of lamps when rotating at 180 revolutions per minute, and which is extremely bad in those types in which the horizontal distribution curve deviates greatly from a circle. Besides the possible error produced by the flicker, the accuracy of a setting is diminished considerably and the eye is rapidly fatigued.

DESCRIPTION OF APPARATUS.

In order to determine definitely the errors due to distortion of the filament and to flicker, an investigation has recently been carried out in the Photometric Laboratory of the Bureau of Standards. A pair of rotating mirrors, Fig. 1 (frontispiece), somewhat similar to those described above as in use at the Reichsanstalt, but with one essential difference, were constructed in the instrument shop of the Bureau. Instead of mounting the lamp in a stationary position between the mirrors, the apparatus was so designed that the lamp could be rotated about its axis quite independently of the rotation of the mirrors. This arrangement obviated the necessity of a separate determination of the coefficient of reflection of the mirrors, which is liable to error unless the mean horizontal intensity of the standard lamp employed is obtained by the point to point method from a very large number of readings.

By designing the apparatus in such a manner that the mirrors and lamp could be rotated independently a substitution method could be employed and the two elements, bending and flicker, could be separately determined irrespective of the reflection coefficient of the mirrors. The assumption is made, of course, that the coefficient of reflection remains the same independent of the speed of rotation. This assumption was subsequently verified by direct experiment. A lamp with an approximately uniform horizontal distribution curve was placed in a stationary position in the socket and the mirrors were rotated first at the lowest possible speed to eliminate flicker, which was about 300 revolutions per minute, and then at 900 revolutions per minute. The mean photometric settings under the two conditions agreed to within 0.1 per cent, which was less than the possible error of observation.

Since the constancy of the reflection coefficient was of such importance in the experiments, great care was exercised in the design of the mirror system to insure rigidity. Fig. 2 is a vertical section of the mirror system through the axis of rotation. The metal plates, *A*, on which the mirrors were mounted, were quite heavy, tapering from a thickness of 6 mm at the outside end to a thickness of 10 mm at the junction with the hub. These plates moreover, were reenforced by longitudinal and transverse webs which added greatly to the rigidity of the mirror supports. Over the surface of each plate a thin layer of plaster of Paris was spread in which the mirror, *M*, was allowed to settle. This formed a hard surface in contact with the mirror at every point and thus prevented any distortion of the mirror under the action of centrifugal forces. Four small bolts, *B*, through each mirror, prevented a possible slipping of the mirrors from the plaster cast.

The frames of the two mirrors were joined at the top and bottom by stay rods, *C*, from two of which a small disk, *D*, was suspended to prevent the direct light in the direction of the tip of the lamp from reaching the photometer. The angle between the two mirrors was such that for a definite distance from the photometer the rays of light emanating from the center of the lamp and normal to the

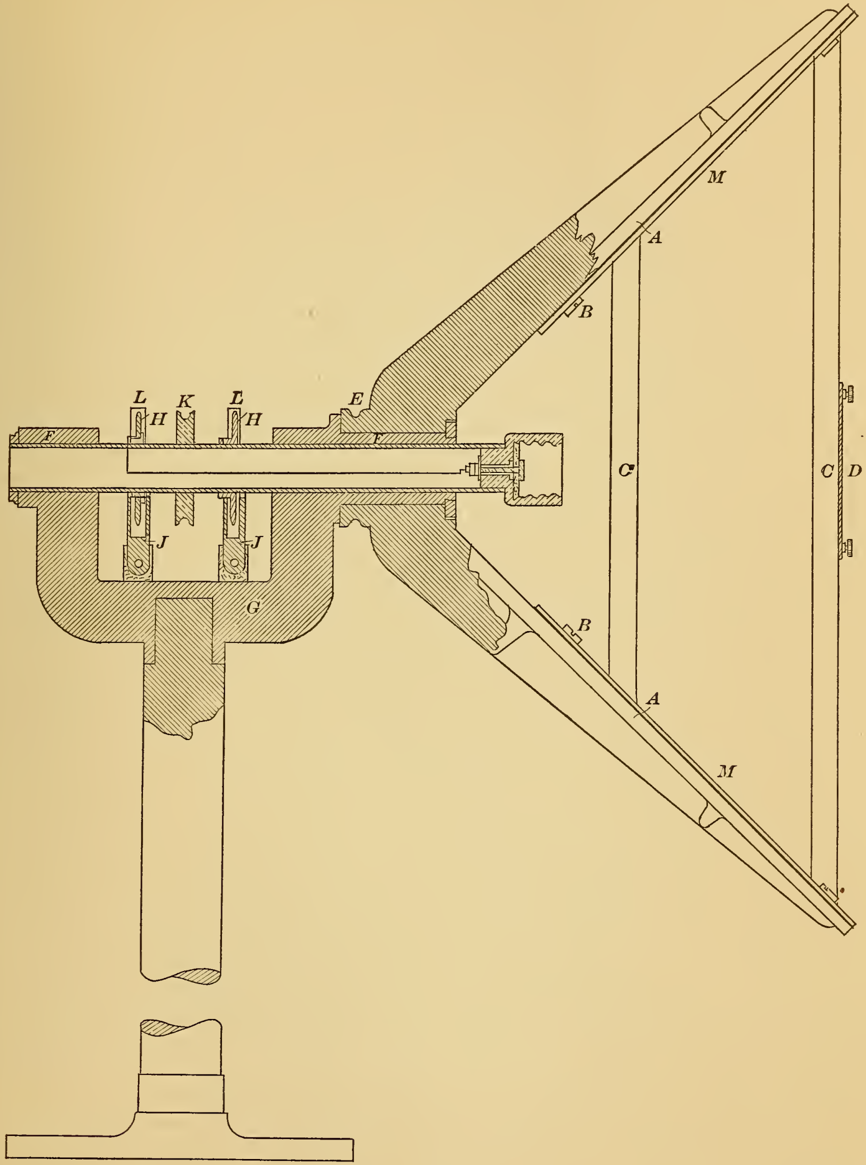

Fig. 2

Vertical Section of Rotating Mirrors through Axis of Rotation.

axis of the lamp were reflected at the mirrors and were incident on the photometer screen at its center.

The entire mirror system was driven by a round belt operating in a grooved pulley, *E*, on the hub. The bearing surface on which the hub rotated was turned down from a boss, *F*, on one arm of a U-shaped casting, *G*. By drilling through the bosses, *FF*, on the two arms of the casting a bearing surface for the shaft carrying the lamp socket was formed. The terminals of the socket were connected to amalgamated copper disks, *H*, mounted on the shaft, and these disks rotated in mercury cups, *J*, supported on the casting. The lamp was driven by means of a grooved pulley, *K*, mounted between the copper disks. This design allowed quite independent rotation of the mirrors and the lamp about the same axis, the mercury contacts preventing any variable resistance when the lamp was

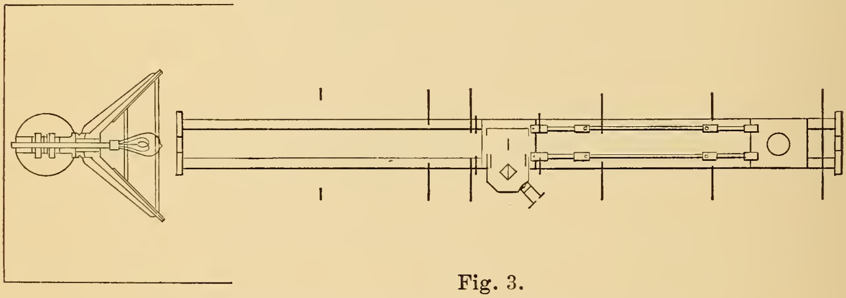

Fig. 3.

Diagrammatic Sketch of Rotating Mirror System in Position.

rotating. Shields, *L*, were constructed over the mercury cups to prevent excessive throwing of mercury at high speeds of rotation of the lamp. The U-shaped casting carrying the mirror system and the lamp was supported on a steel shaft which was sweated into a large base. The base was screwed down to a heavy oak board, which was clamped on a table at the end of a photometer bench in such a position that the axis of rotation of the mirrors and lamp coincided very closely with the photometric axis. Fig. 3 is a diagrammatic sketch showing the rotating mirror system in position.

Since with the introduction of mirrors the system of diaphragms commonly used on the photometer was inadequate, a house of black velvet was built around the rotating mirrors so that no light could reach the photometer except that directly reflected from the mirrors and the stray light reflected from the black velvet.

The electrical measurements were made by the aid of a potentiometer. In the photometric measurements the substitution method was used, the comparison lamp remaining at a fixed distance from the photometer and moving with it as a single system.

GENERAL METHOD.

In order to determine with the above apparatus the change in mean horizontal intensity due to bending and flicker, both separately and together, the following series of measurements was made on lamps of a number of different types: (*a*) mirrors rotating at a sufficiently high speed to eliminate flicker (in some cases over 800 revolutions per minute), lamp at rest; (*b*) mirrors rotating at 180 revolutions per minute, lamp at rest; (*c*) mirrors at rest, lamp rotating at 180 revolutions per minute; (*d*) mirrors at rest, lamp rotating at 500 or 600 revolutions per minute; then the entire cycle in the reverse order. In all the measurements in order that the observer at the photometer should not be influenced by his readings a second observer noted and recorded the settings on the bar, and the observer at the photometer was not informed of the results until the entire series of measurements had been completed.

If there were no error due to bending, and no error due to flicker, all four of the above sets of readings would be the same. A comparison of (*a*) and (*b*) shows the effect of flicker separated from that of distortion of the filament; a comparison of (*a*) and (*c*) shows the combined error due to flicker and bending at 180 revolutions per minute. This combined error is that which occurs in practice when measurements are made on lamps rotating at 180 revolutions per minute. A comparison of (*b*) and (*c*) shows the effect of bending at 180 revolutions per minute; and a comparison of (*a*) and (*d*) shows the effect of bending at 500 or 600 revolutions per minute.

In all, ten different types of incandescent lamps were studied, as follows:

A– 50	volt,	16	cp, horseshoe filament lamp.
B–220	"	16	" double oval anchored filament lamp.
C–110	"	16	" oval anchored filament lamp.
D–110	"	16	" double carbon filament lamp.
E–110	"	16	" double flattened coil filament lamp.
F–110	"	16	" double round coil filament lamp.

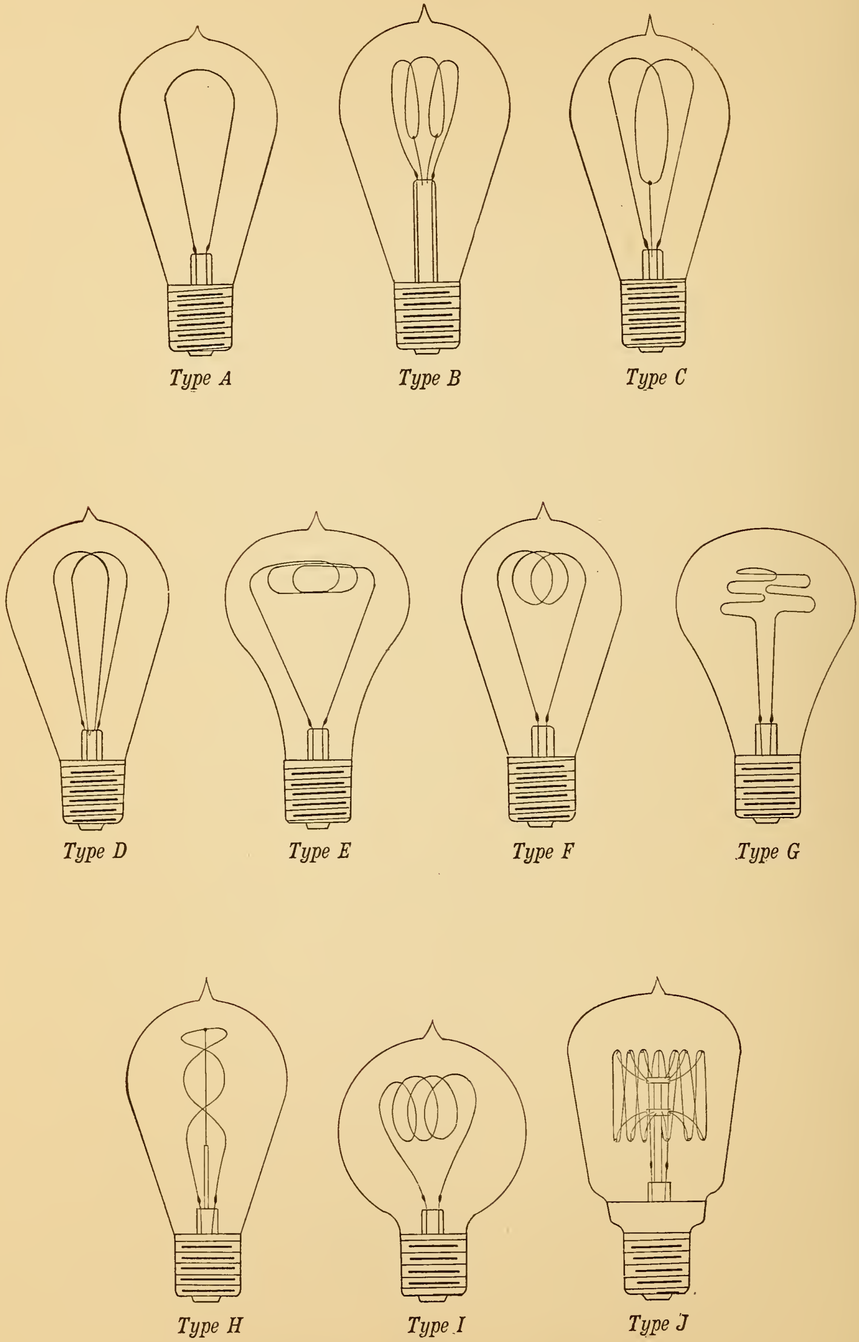

Fig. 4.

Different Types of Lamps Investigated.

G–110 volt, 16 cp, downward light filament lamp.
H–110 " 16 " spiral filament lamp.
I–118 " 16 " round bulb, helical filament lamp (concentrated carbon deposit).
J–110 " 22 " tantalum lamp.

The forms of the filaments of these ten different types of lamps are shown in Fig. 4.

The results of the investigation can be summarized under the two heads—"Effect of Bending," and "Effect of Flicker."

EFFECT OF BENDING.

At speeds of 500 or 600 revolutions per minute very appreciable changes in the mean horizontal intensity due to bending were observed, in some cases the effect being to decrease the mean intensity, in others to increase it. The greatest difference due to bending was observed in a double carbon filament lamp which had accidentally been rotated for a few seconds at about 800 revolutions per minute and was afterward brought to a speed of about 600 revolutions per minute. This lamp showed a decrease in mean horizontal intensity of about 2 per cent, but after the lamp was stopped rotating it was noticed that one loop of the filament had touched the bulb and had sealed itself there, remaining permanently in that position. Except for this one lamp the results on the effect of bending at a speed of 550 revolutions per minute for the lamps of the different types studied are summarized in Table I. In the first column the type of lamp is given. The second column contains the individual determinations of the effect, (+) indicating that the effect of rotation was to increase the mean horizontal intensity, (–) that the effect was to decrease it. In the third column are given the means of the individual determinations for each type of lamp.

It is seen from Table I that the greatest change in mean horizontal intensity due to a bending of the filament occurred in the double carbon type of filament in which on the average a decrease of 0.9 per cent, was observed. The greatest increase was 0.8 per cent, which was the mean value found for the downward light type of filament. The oval anchored filament showed a decrease of about 0.5 per cent. At a speed of 180 revolutions per minute the change in mean horizontal intensity due to bending was so small for all types of lamps studied as to lie within the range of experimental error, which was probably several tenths of one per cent.

TABLE I.

Effect of Bending.

Type of lamp	Change in mean horizontal intensity	Means	Type of lamp	Change in mean horizontal intensity	Means
A	±0.0%	±0.0%	E	+0.3%	
B	−0.1	−0.1		+0.6	
C	−0.6			+0.4	+0.2%
	−0.3			±0.0	
	−0.4			±0.0	
	−0.3	−0.5		±0.0	
	−0.7		F	−0.2	
	−0.4			+0.1	±0.0
	−0.6			±0.0	
D	−0.7		G	+0.9	+0.8
	−1.1	−0.9		+0.6	
	−0.8		H	−0.1	
				+0.1	−0.1
				−0.3	

The change in mean horizontal intensity produced by bending is to be explained by the difference in the mean effective length of the filament in the two cases. A straight vertical filament would have a maximum intensity in the horizontal plane. If the filament were bent due to spinning, its effective length would be decreased; or, looking at the question in another way, the light in the horizontal plane would be emitted at some angle, θ, to the normal, and consequently the intensity for each element of filament would be decreased in the ratio of cos θ:1. For a straight vertical filament we could be certain that the effect of bending would be to decrease the mean horizontal intensity. In the case of complex filaments, if the filament simply bent as a whole, we could be certain that the bending in all types of lamps would increase or decrease the mean horizontal intensity depending upon whether the mean horizontal intensity is less or greater than the mean latitudinal intensity on either side of the horizontal. Since, however, superimposed upon the effect of bending of the filament as a whole is the effect due to the relative bending of the parts of the filament, it is difficult to state à priori the direction or magnitude of the total effect. It is significant, however, that the two types of lamps in which the greatest increase and greatest decrease in mean horizontal intensity were observed are those in which the mean horizontal intensity is notably a minimum, and a maximum, respectively.

The tantalum lamp showed a queer effect which it is interesting to note in passing. Although one might expect a decrease in mean horizontal intensity on rotation, no such effect was found, the observations indicating a slight increase. The magnitude of the effect was small, but since in every one of a number of determinations the sign of the effect was the same it is probable that there is a real increase in mean horizontal intensity. The peculiar feature to which attention should be called, however, is the apparent increase in resistance of tantalum lamps on rotating them. At least a half dozen lamps were tested, each of them several times, and without exception on rotating the lamps the current decreased, although the voltage was maintained constant. The amount of change was not the same for all lamps, nor indeed the same at different times for any one lamp, but ranged from 1 per cent to only a few tenths of 1 per cent.

EFFECT OF FLICKER.

According to Talbot's law, if the retina is excited with intermittent light recurring periodically, and if the period is sufficiently short, a continuous impression will result which is the same as that which would result if the total light received during each period were uniformly distributed throughout the whole period. Assuming this law, which has recently been investigated[5] by one of the present writers and shown to be true for the extreme case of a rotating sectored disk, the method of rotating the lamp would give the true mean horizontal intensity if the speed of rotation were sufficiently high to eliminate all flicker, provided there is no distortion of the filament. Matthews, by direct experiment,[6] showed Talbot's

[5] Bulletin of Bureau of Standards, **2**, p. 1; 1906.

[6] Physical Review, **6**, p. 55; 1898.

law to hold under these conditions. But in almost all practical cases it is impossible to rotate the lamp at a sufficiently high speed to eliminate flicker without introducing errors due to distortion of the filament. Since we are not justified in extrapolating from Talbot's law to include cases in which there is a perceptible flicker, it is necessary to test by experiment whether or not the eye integrates a flickering light.

The most surprising results of the investigation were found in studying the effect of flicker on the apparent mean horizontal intensity of the different types of lamps. The common oval anchored type of filament shows comparatively little flicker when rotating at 180 revolutions per minute, and consequently, when small differences between sets (*a*) and (*b*) were found with this type of lamp they were ascribed to observational errors, since the difficuly of setting with a flickering light is much greater than when the fields of view in the photometer appear steady. When, however, with other types of lamps differences amounting to 4 per cent occurred it became evident that the effect was not to be ascribed to the probable error of observation, for every one of 5 or 10 readings with the mirrors at high speed would lie entirely without the limits of the readings at 180 revolutions per minute. Moreover, with different observers the effect of the flicker was found to be quite different. Thus, while one of us read the flickering light about 4 per cent too high, the other read about 3 per cent too low, a difference of 7 per cent. Moreover, each observer persisted in his habit, so that when, about this time, a number of lamps submitted for test were measured it was subsequently found on looking over the results that while the values obtained by the two observers agreed excellently for most lamps, there were large differences, always in the same direction, for lamps in which there was a disagreeable flicker.

Other observers were tried, of whom some were experienced in photometry, and all were experienced in other lines of physical investigation, but in almost every case there was a decided preference for either higher or lower values with the flickering light. All of the measurements referred to above were made with a Lummer-Brodhun contrast photometer, and in every case there was a color match on the two sides of the screen. Subsequently a number of measurements were made by three observers with a very good Leeson disk furnished the Bureau by the Electrical Testing Laboratories of New York. Strangely enough, with this photometer each of the three observers changed the sign of his error as compared with his measurements on the Lummer-Brodhun photometer. Of course, the error of observation with a badly flickering light, such as is obtained with a double flattened or a downward light filament, is quite large, but there is no doubt in our minds that the error due to flicker is a very definite one, characteristic of the individual. It would not seem, however, that the effect is a true physiological effect in the same sense as Talbot's law for nonflickering illuminations. It is not that different eyes integrate a flickering light differently, but rather that they do not integrate at all. The explanation would seem to be this: Due to the inability of the eye to integrate a flickering light, an infinite number of different independent intensities are perceived. On any one of these a setting could be made, but having chosen some one point in the series of fluctuating intensities the criterion persists, and the more observations the individual makes the more probable it is he will continue to set in the same way. One of the observers previously mentioned was conscious of two different criteria by either of which he was able to make consistent settings. According to one criterion he set too low; according to the other, too high.

Apart from the explanation of the effect of flicker the important fact which the investigation makes clear is, that in the case of lamps in which the horizontal distribution curve deviates considerably from a circle so that a bad flicker results when the lamp is rotated at 180 revolutions per minute, some other method for determining the mean horizontal intensity must be employed. Otherwise, large errors amounting to many per cent may arise.

It is impossible to state what the errors may be because they depend entirely upon the individual. It may be that some observers may obtain erroneous settings with such a small flicker as that incident to an oval anchored filament lamp when rotating at 180 revolutions per minute, but it is very improbable that any large errors would be made except with a badly flickering light, such as occurs with the double flattened or downward light lamps at 180 revolutions per minute. It is true that these two types of lamps in which the flicker is most pronounced are not rated at mean horizontal intensity by their manufacturers, but since in accordance with many specifications all lamps are purchased on this one basis, it becomes necessary to determine the mean horizontal candlepower of these lamps. Moreover in some other types of lamps which are usually rated on a basis of mean horizontal intensity, such as round bulb lamps with helical filaments, bad flickers occur and consequently large errors may result in the determination of the mean horizontal intensity by rotating the lamp at 180 revolutions per minute. The great danger in making measurements with a flickering light is that the observer has no easy way of determining the magnitude of the error which he may be making, and consequently there is always an uncertainty in the results.

Since none of the filaments of the different types of lamps investigated suffered any apparent injury at the high speed of 500 revolutions per minute and the effect of bending was in no case more than 1 per cent, it was thought desirable to investigate possible errors incident to the determination of mean horizontal intensity by rotating the lamps at a speed double that in use at present, i. e. at 360 revolutions per minute. Before doing this, however, some experiments were made on the mechanical effects of such a speed on lamps the filaments of which had drooped, due to prolonged burning in a horizontal position. Three 50 cp, 110 volt lamps, each of which had burned several hundred hours and had fallen in intensity below 80 per cent of its initial value, showed excessive drooping. In one lamp the filament was not more than 3 or 4 mm from the bulb; in each of the other two the filament was about 5 or 6 mm from the bulb. These lamps, which had long heavy filaments and which had given evidence of their susceptibility to bending under the influence of mechanical forces by their excessive drooping under the action of gravity, would seem to represent the most unfavorable case for a high speed rotation. The lamp in which the filament was within 4 mm of the bulb stood a speed of 300 revolutions per minute. At about 350 or 400 revolutions per minute the filament touched the bulb and melted it at the point of contact, allowing air to enter, which caused the lamp to burn out. The other two lamps were rotated at increasing speeds up to about 650 or 700 revolutions per minute before the filament touched the bulb, in every case the result of the contact being the same. Since these lamps would probably have been rejected on most specifications before dropping to 80 per cent because of the excessive drooping, it would seem that a speed of 360 revolutions per minute would be quite safe in all practical cases. Although measurements made on different types of lamps indicated no determinate permanent change in the horizontal intensity, the effect of continued burning, while rotating at a speed of 360 revolutions per minute, was not investigated.

After showing that a speed of 360 revolutions per minute would probably produce no serious mechanical difficulties, samples of those types of lamps in which the effect of flicker is most pronounced and of those in which the bending is greatest, were measured at that speed in order to see whether the errors due to flicker could be obviated without introducing prohibitive errors due to bending. It was found that in several types of lamps the flicker was still so pronounced that differences of several per cent were obtained. It was therefore concluded that either a higher speed must be used or some other method employed.

Since it would not seem feasible to increase the speed much above 360 revolutions per minute an attempt was made to devise some other method by which an increased accuracy could be obtained. The following considerations suggest a method which was put into practice and found to be quite satisfactory: The error due to the flicker is a function not only of the speed, but of the difference between the maxima and the minima of the flickering light. No matter how great this difference the flicker will cease at sufficiently high speeds, but if the difference is small the error due to the flicker will be relatively small at all speeds and will entirely disappear when the speed has been increased even to a moderate value. In those types of lamps in which the error due to flicker was found to be large the differences between the maxima and the minima are great, amounting in one type to 70 per cent of the mean value, so that if in some way the extreme fluctuations in the intensity of illumination on the photometer screen could be lessened the resultant error would be diminished.

Except in the case of anomalous lamps extreme differences in intensity in different directions are not due to bad centering of the filament or to any effect connected with a displacement of the effect-

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ive center of radiation from the axis of rotation; fluctuations due to these effects would in general be relatively small. The irregularity of the horizontal distribution curve is due primarily to the shape of the filament, and consequently, except for sharp reflections from the bulb or sharp shadows produced by the obscuration of parts of the filament, the maxima lie 180° apart, and in general the minima lie approximately at right angles to the maxima. Hence, if to the illumination produced by the direct light from the lamp on the photometer screen, an illumination produced by light emitted in a direction normal to the photometric axis be added, the resultant illumination will show a relatively small variation on turning the lamp. If the lamp is rotated at the customary rate of 180 revolutions per minute the flicker will be much less disagreeable and productive of error than that incident to the direct light alone when the lamp is rotated at double this speed.

In order to accomplish the above result in practice a single stationary mirror (Fig. 5) was placed in such a position that light

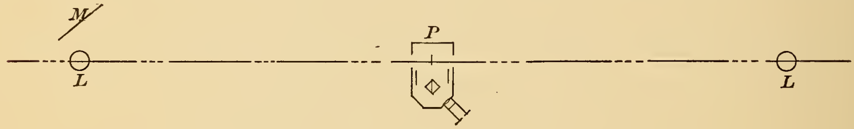

Fig. 5.

Sketch Showing Stationary Auxiliary Mirror in Position.

emitted at right angles to the photometric axis was reflected by the mirror and was incident on the screen in conjunction with the direct light from the lamp. Measurements made on the downward light lamp at 180 revolutions per minute by the two observers who had previously differed by about 7 per cent at this speed, showed an agreement well within the range of error of observation, which was probably less than one-half of 1 per cent. In a similar way the error due to flicker in all types of lamps in which the maxima and minima lie approximately at right angles to each other would become quite small even at a speed of 180 revolutions per minute. The results of the investigation at the Bureau show that a single mirror is sufficient for the types of lamps studied, but it is possible that other arrangements of mirrors might be devised that would also eliminate flicker.

The principal point to be emphasized is the general idea of combining the rotating lamp method with that of stationary mirrors.

In the subsequent paragraphs, however, the theoretical discussion will be confined to the case of a single mirror.

One or two round bulb lamps which had burned to 80 per cent in life test, and which were unique in having most of the carbon deposit concentrated on a small part of the bulb, showed bad flickers even with the addition of the mirror. This was to be expected, as measurements showed that, due to the absorption by the concentrated carbon deposit, there was a difference of 40 per cent between the intensity in the direction of the deposit and in a direction 180° from the deposit. In one direction the intensity was a maximum, in the other a minimum. With such anomalous lamps a very high speed must be used or some other method employed if accurate results are desired.

Having shown that the use of a single mirror diminishes the flicker to such an extent for all types of lamps investigated that the error due to flicker is negligible in commercial photometry, it only remains to show that the application of the mirror is entirely practical, taking into consideration the difference in path between the direct ray from the lamp and the ray by way of the mirror. Assuming that the two lamps are at fixed positions, and that the substitution method of measurement is employed, calculations were first made to determine what the error would be if the simple law of inverse squares were used and the distance to the effective center of radiation assumed to be intermediate between the distance of the lamp and the distance of its image from the screen. The candlepower values at definite distances on either side of the center (16-cp position) were computed on this basis and compared with the true candlepower values for the corresponding points, assuming the reflection coefficient of the mirror to be 0.85, and assuming the center of the mirror 15 cm from the axis of the lamp, measured in a direction perpendicular to the photometric axis. The results showed that for a bar 250 cm long the errors which would occur in measuring lamps ranging from about 4 cp to 150 cp would in no case be much greater than 1 per cent, but that with an increasing range of candlepower, particularly in the direction of lower candlepower values, the errors would rapidly increase. Since it is quite simple, however, to compute a rigorous candlepower scale which will be true for any position, assuming that the reflection coefficient of the mirror is a known constant, no further investigation of the preceding method was made, such as the effect of a change in the reflection coefficient of the mirror. In the following paragraphs the rigorous method will be investigated in detail.

Let O, O' (Fig. 6) be the centers of the two lamps at a fixed distance, $2a$, apart, and let O'' be the image of O in the mirror BCD. The center, C, of this mirror is at a distance d from the lamp O, and lies on the line through O perpendicular to the photometric axis OO'. The mirror is turned in such a direction that the ray OC is incident on the photometer screen at its center when the photometer screen is at A, midway between the two lamps. For any other

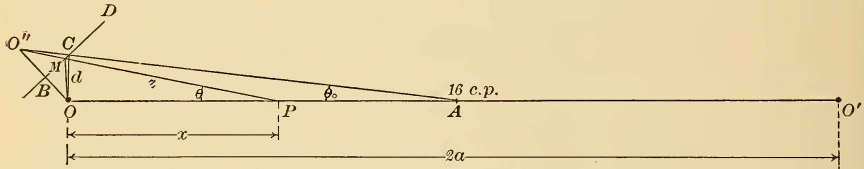

Fig. 6

position, P, of the photometer screen the path of the reflected ray will be OMP.

If we let J and J' be the intensities of the two lamps at O and O', r the reflection coefficient of the mirror, x the distance from the lamp O to the photometer screen, z the distance from the image O'' to the photometer screen, and θ the angle of incidence of the reflected beam of light on the photometer screen, the equation of photometric balance at any position, P, will be

$$\frac{J}{x^2}+\frac{Jr\cos\theta}{z^2}=\frac{J'}{(2a-x)^2} \qquad (1)$$

z^2 can be expressed in terms of x as follows:

$$z^2=\overline{OO''}^2+x^2-2\,\overline{OO''}x\cos POO'' \qquad (2)$$

and if θ_0 is the value of θ corresponding to the middle point, A, of the photometer bar,

$$\overline{OO''}=2\overline{OB}=2\overline{OC}\cos COB=2d\cos\frac{1}{2}(90^\circ-\theta_0) \qquad (3)$$

where θ_0 is determined by the equation

(4) $$\tan \theta_0 = \frac{d}{a}$$

Moreover

(5) $$\text{angle } POO'' = 90° + \text{angle } COO''$$
$$= 90° + \frac{1}{2}(90° - \theta_0)$$

(6) $$= 135° - \frac{1}{2}\theta_0$$

Hence, all the terms in the right hand member of equation (2) can be expressed in terms of the variable, x, and the constants, a and d. Cos θ is determined from the sine relation,

(7) $$\sin \theta = \frac{\overline{OO''}}{z} \sin POO''$$

in which the right hand member is a function of x, a, and d (equations (2), (3), (4), and (6)).

Having chosen the fixed distance, $2a$, between the two lamps, and the fixed distance, d, from the lamp O to the center of the mirror, $\overline{OO''}$ and the angle POO'' are determined and remain constant in the calculation of the candlepower values corresponding to different distances, x. From equation (1), which may be put in the form

(8) $$\frac{J}{J'} = \frac{1}{(2a-x)^2} \frac{x^2z^2}{rx^2 \cos \theta + z^2}$$

the ratio of J to J' may be calculated for different distances, x, and by choosing a value of J' which will bring a balance at the middle point of the bar ($x=a$) for $J=16$ cp, the candlepower values of J corresponding to other values of x may be readily computed, and the candlepower scale plotted.

In order to show how this equation works out in practice let us apply it to an actual case. On the photometer bench employed at the Bureau for commercial testing, the two lamps O and O' are 250 cm apart. Since the diameter of the largest bulb lamps that are ordinarily met with in the phometry of incandescent lamps does not exceed 13 cm, the distance from the lamp to the center of the mirror may be taken as $d=15$ cm, in which case the image of the lamp in the mirror will in no case be obscured by the lamp itself, provided the photometer screen is not brought below the position corresponding to 2 cp.

If we put $2a=250$ cm, and $d=15$ cm, we obtain, on substituting in equation (4);

$$\tan\theta_0=\frac{d}{a}=\frac{15}{125}$$
$$=6^\circ 50' 34'' \tag{9}$$

From equation (3)

$$\overline{OO''}=2d\cos\frac{1}{2}(90^\circ-\theta_0)$$
$$=2\times 15\cos\frac{1}{2}(90^\circ-6^\circ 50' 34'')$$
$$=22.44 \text{ cm} \tag{10}$$

From equation (5)

$$\text{angle } POO''=135^\circ-\frac{1}{2}\theta_0$$
$$=135^\circ-\frac{1}{2}(6^\circ 50' 34'')$$
$$=131^\circ 34' 43''$$

Substituting these values in equation (2) we get

$$z^2=\overline{OO''}^2+x^2-2\overline{OO''}\,x\cos POO''$$
$$=503.6+x^2+29.78x$$
$$=(x+14.89)^2+281.9 \tag{12}$$

Therefore equation (8) becomes

$$\frac{J}{J'}=\frac{1}{(250-x)^2}\,\frac{x^2\,((x+14.89)^2+281.9)}{rx^2\cos\theta+(x+14.89)^2+281.9} \tag{13}$$

where $\cos\theta$ is obtained from the relation (equation (7))

$$\sin\theta=\frac{\overline{OO''}}{z}\sin POO''$$

$$=\frac{22.44}{\sqrt{(x+14.89)^2+281.9}}\sin 131^\circ 34' 43'' \tag{14}$$

By assigning different values to x in equations (13) and (14) the ratio $\frac{J}{J'}$ may be computed for any reflection coefficient, r. Let us make $r=0.85$, which represents approximately the average value for good silver-back mirrors, and let us compute the ratio $\frac{J}{J'}$ for the distances, $x=65$, 95, 125, 155, and 185 cm. The results of the computation are given in the sixth column of Table II. If now a 16 cp standard is to balance against the comparison lamp at the middle point of the bar, $x=125$ cm, the required intensity J' of the comparison lamp is immediately obtained by putting $J=16$ for $x=125$ cm and computing the value of J' from the equation

$$\frac{J}{J'}=0.6009 \tag{15}$$

from which $J'=26.63$ cp.

By giving J' this value in the ratio $\frac{J}{J'}$ at other distances, the candlepower values corresponding to the respective distances $x=65$, 95, etc., are obtained. These are given in the seventh column of Table II.

TABLE II.

Values of $\frac{J}{J'}$ and J for Different Reflection Coefficients, r, and Different Distances, x.

x	r=0.75		r=0.80		r=0.85		r=0.90		r=0.95	
	$\frac{J}{J'}$	J	$\frac{J}{J'}$	J	$\frac{J}{J'}$	J	$\frac{J}{J'}$	J	$\frac{J}{J'}$	J
65	0.08425	2.138	0.08250	2.145	0.08082	2.152	0.07922	2.159	0.07767	2.165
95	0.2437	6.185	0.2381	6.192	0.2328	6.199	0.2272	6.206	0.2228	6.212
125	0.6305	16.00	0.6153	16.00	0.6009	16.00	0.5870	16.00	0.5739	16.00
155	1.6480	41.82	1.6072	41.79	1.5684	41.76	1.5313	41.74	1.4960	41.71
185	4.952	125.68	4.827	125.53	4.708	125.38	4.595	125.24	4.487	125.09

Of course, in plotting a candlepower scale the value of J for a great many more distances, x, must be used, but the values given in Table II indicate the method to be employed, and cover the range of candlepower likely to be used.

It is interesting now to determine what change in the candle-power scale is necessary corresponding to a different reflection coefficient. This determination is important for two reasons: (1) it indicates the degree of accuracy required in the initial measurement of the reflection coefficient of the mirror to be used, and (2) it indicates what change may take place in the reflection coefficient of the mirror after it has been installed, without necessitating a new candle-power scale. The results of this investigation are given in Table II. A study of this table shows that if the reflection coefficient of the mirror is 0.80 or 0.90 instead of 0.85, i. e. if it is 5 or 6 per cent different from that used in computing the scale, the extreme error which occurs at the lower limit of the scale, i. e. at $J=2.15$ cp, is only 0.3 per cent. Even if the reflection coefficient is 10 or 12 per cent different, the error is negligible except at the extremely low value of $J=2$ cp where it may reach 0.7 per cent. At this position of the photometer, however, larger errors may occur from other sources.

In the above discussion it was implied that the substitution method of measurement is employed. Since this method is almost universally used there is no need of considering any other. In this method a standard 16-cp lamp is placed at O, and the voltage on the comparison lamp at O' is changed until a balance is obtained at the 16-cp point at the middle of the bar. In the employment of an auxiliary mirror the illumination is almost doubled, so that the comparison lamp must have an intensity of 25 or 30 cp, being different for mirrors of different reflection coefficients. If, however, the comparison lamp is made to balance the 16-cp standard at the middle of the bar, no matter what the reflection coefficient may be, the readings on a scale computed in terms of $r=0.85$ will be correct near the 16-cp point, and the error at 2 cp will only be 0.7 per cent provided the reflection coefficient is not more than 10 or 12 per cent different from 0.85.

It has been assumed in the above discussion that the reflection coefficient, r, remains constant for all positions of the photometer screen. There are two reasons, however, why this may not be true: (1) the mirror must be approximately plane, and must have approximately the same coefficient of reflection at different parts of the surface in the region employed; (2) it must be shown that the angle of incidence corresponding to different positions of the photometer screen does not change sufficiently to produce a difference in r. Measurements made on plate glass mirrors indicated that the error due to the latter cause would probably not be larger than 0.3 or 0.4 per cent, even over the entire range of the scale. Moreover, both this error and that which would result if the mirror were not plane or uniform become entirely negligible if the photometer is always kept near the center of the scale. This is accomplished in the photometer at the Bureau by the use of a rotating sectored disk, calibrated for 32, 50, 64, and 100 cp, with corresponding candle-power scales on a revolving drum. To obtain values lower than 16 cp, such as 8 and 4 cp, the disk is placed on the other side of the photometer, between it and the comparison lamp. This arrangement was employed before the investigation of the auxiliary mirror was undertaken as being much less liable to error than an extended range of candlepower values on a single scale.

With reasonable care in the selection of the mirror, however, there is no need to fear any large error due to any of the above possible sources of error even if a single scale is used. The worst errors that could occur would be those in measuring lamps of 2 or 3 cp, and since such measurements are seldom required, and the accuracy demanded is never high, the single mirror method can be used with entire confidence.

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