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# Dynamic Green's Functions of an Infinite Plate - A Computer Program

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, *Secretary*  
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director*



# Dynamic Green's Functions of an Infinite Plate - a Computer Program

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## Introduction

This report is a FORTRAN program to compute the Green's functions of an infinite plate. The Green's function,  $G_{ij}(\xi, \underline{x}, t)$ , is defined as the  $i$ th component of the displacement at  $\underline{x}$  due to a point force of step-function time dependency acting at  $\xi$  in the  $j$ th direction initiated at  $t=0$ . The Green's function is the fundamental solution of the transient elastic wave propagation problem. In general, the displacement field  $u(\xi, \underline{x}, t)$  at  $\underline{x}$  due to a point force of arbitrary time dependence acting at  $\xi$  can be computed by a convolution integration; i.e.,

$$u_i(\xi, \underline{x}, t) = \int_0^{\infty} G_{ij}(\xi, \underline{x}, \tau) f_j(\tau-t) d\tau. \quad (1)$$

Here,  $G_{ij}$  is the time derivative of  $G_{ij}$  and  $f_j(t)$  is the point force component of arbitrary time dependence acting in the  $j$ th direction (summation over repeated indices is used). Displacement due to point dipoles or couple forces can be represented by the spatial derivatives of  $G_{ij}$ . Displacement produced by a dynamic force distributed over a finite area can also be computed by numerical integration using the Green's function as the kernel of the integral over the finite area.

The basic formulation of the problem and derivation of the solution formulas were reported in (1). The method used is called "ray theory" in the seismological literature. Our derivation was based upon John Willis' new Fourier Inversion method (2). Similar computation results can be found in Reference (3), (4), and (5).

The program was originally written for the analysis of acoustic emission signals. Acoustic emission is a nondestructive testing and monitoring technique in which the detection of the transient stress wave produced by localized deformation or cracks are used to locate, and to assess the criticality of the defects. The theoretical computation is an important link to predict how the acoustic emission waveform evolves through the structure. How the theoretical computations are used in the study of acoustic emission can be found in References (1), (6), (7), and (8).

This computer program is made available mainly for its application to calibrate acoustic emission systems and sensors. By making the present program available, duplication of efforts can be avoided, errors in theory can be checked, and experimental results can be reproduced and verified.

## How to Use the Program

I. The easiest way to run this program is by setting up nondimensional parameters and calling the subroutine GREENFCT. First, the  $x$ - $y$ - $z$  coordinates

are selected by choosing an x-y plane parallel to the plate and the origin at the center of the plate directly underneath the source, i.e., the source is always located at  $\xi=(0,0,0.5)$ ; the x axis is aligned in the direction from the source pointing to the detector. The required nondimensionalized input parameters for the subroutine GREENFCT are:

1. ALPHA = shear wave speed/longitudinal wave speed;
2. XD = x-coordinate of the detector; actual length/thickness of the plate;
3. ZD = z-coordinate of the detector; actual length/thickness of the plate.  
 $ZD = 0.5$  if the source and the detector are on the same side of the plate;  
 $ZD = -0.5$  if they are on opposite sides of the plate.

(Note: the y-coordinate of the detector is always zero because of the way the coordinate system is chosen.)

4. INDEX = ij; subscript of Green's function, integer number 11, 12, 13, 21, 22, 23, 31, 32, or 33. (111, 112, 113, etc. for force dipoles.)
5. TDELTA = sampling time interval in terms of nondimensionalized time unit  
= actual sampling time interval \* shear wave speed/thickness of the plate; and
6. NPOINT = Total number of sampling points to be computed; must be an integer.

The subroutine will return a double precision array DISPL of dimension (NPOINT) which corresponds to the desired  $G_{ij}(\xi, x, t)$  or  $G_{ijk}(\xi, x, t)$  sampled at equal time intervals. However, the displacements corresponding to dipoles or couples (i.e.,  $G_{ijk}$ ) are for linear ramp time dependency input. Differentiating the returned DISPL with respect to time once will give the proper Green's function due to step time dependency input.

A simple program calls GREENFCT for given nondimensionalized input parameters together with the output results is included in Appendix A; a program that prompts the user for input parameters of arbitrary physical units and returns displacement in physical units is included in Appendix B. The complete listing of the subroutines is in Appendix C.

II. Another way to use this program is by calling GREENSUB which is basically a simple function: for given one time T, it returns a displacement DISPL.

However, there are two preparatory steps that must be taken in the main program:

(1) RAYTIME (NRAY, 3), the time of arrival table, TA(NRAY) and CN(NRAY, 3), two working arrays, should be dimensioned.

(2) The subroutines INIT and TIMEARRI must be called first.

The subroutine GREENFCT may serve as an example of how to use the subroutine GREENSUB.

III. In addition to the two subroutines mentioned above, there are three subroutines which may be called independently and the user may find them useful in checking experimental or theoretical results.

(1) EPIDIS computes the vertical displacement at the epicenter for given ALPHA and T.

(2) SFWAVE computes the surface wave, which corresponds to the Green's function of a semi-infinite space.

(3) TIMEARRI computes the time of arrival for various ray paths.

Comments in these three subroutines serve as instructions about how to use these subroutines.

#### Some Remarks

I. There are two machine dependent constants which are used in the two numerical integration routines DGLQI and GLIST. EPMACH is the largest relative spacing. UFLOW is the smallest positive magnitude. See Reference (9) for details about how to set these constants for different computers.

II. The program uses double precision complex arithmetic. In order to run on those FORTRAN compilers lacking double precision complex function libraries, the user may have to modify the program to single precision arithmetic.

III. The ray method is an exact solution for the Green's function in the time domain produced by summing the contribution from successive arrivals of reflected rays. The numerical computation is accurate if the source and detector are near each other (say less than ten plate thicknesses) and if the maximum time is less than ten dimensionless units. If the sensor and detector are far apart or the maximum time is large, the number of rays arriving at approximately the same time may be too large to compute in a reasonable time and the accumulated error may grow.

IV. The current program is limited to the test configuration where the sensor and detector are on the surface of the plate. The displacement computed for force dipole (i.e., INDEX = 111, 112, etc.) are for point dipole of linear ramp time dependency. Differentiation with respect to time once of the displacement will produce proper Green's functions due to step time dependency (See Reference 10).

#### Acknowledgement

The author would like to thank Dr. David Kahaner of the Scientific Computing Division, NBS, for supplying the integration subroutines DGLQ and GLIST.

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## Appendix A

### Sample Program and Results



```
C*****
C
C      PROGRAM TSGREEN
C
C      TEST PROGRAM TO COMPUTE GREEN'S FUNCTION OF A PLATE
C
C      DOUBLE PRECISION DISPLC(256)
C
C      WRITE(5,*)'TYPE INDEX,ALPHA,XD, AND ZD'
C      READ(5,*) INDEX,ALPHA,XD,ZD
C      INDEX=33
C      ALPHA=0.59902
C      XD=1.0
C      ZD=-0.5
C      WRITE(5,*) 'TYPE DELTAT AND NUMBER OF POINTS'
C      READ(5,*) TDELTA,NPOINT
C      TDELTA=0.01
C      NPOINT=256
C
C      CALL GREENFCT(ALPHA,XD,ZD,INDEX,TDELTA,NPOINT,DISPLC)
C
C      DO 1000 I=1,256
C          TIME = 0.01*REAL(I)
C          WRITE(6,1001)  TIME,DISPLC(I)
1000 CONTINUE
1001 FORMAT(2X,'TIME = ',F8.3,',   DISPLACEMENT = ',D21.14)
      STOP
      END
```



TIME = 0.550, DISPLACEMENT = 0.00000000000000D+00  
TIME = 0.560, DISPLACEMENT = 0.00000000000000D+00  
TIME = 0.570, DISPLACEMENT = 0.00000000000000D+00  
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| TIME = | 2.480, | DISPLACEMENT = | 0.12574953844797D+01 |
| TIME = | 2.490, | DISPLACEMENT = | 0.12651217887352D+01 |
| TIME = | 2.500, | DISPLACEMENT = | 0.12729111430751D+01 |
| TIME = | 2.510, | DISPLACEMENT = | 0.12808706969625D+01 |
| TIME = | 2.520, | DISPLACEMENT = | 0.12890055553986D+01 |
| TIME = | 2.530, | DISPLACEMENT = | 0.12973216346622D+01 |
| TIME = | 2.540, | DISPLACEMENT = | 0.13058241075455D+01 |
| TIME = | 2.550, | DISPLACEMENT = | 0.13145206649727D+01 |
| TIME = | 2.560, | DISPLACEMENT = | 0.13234166514682D+01 |
| TIME = | 2.080, | DISPLACEMENT = | 0.95928037273261D+00 |
| TIME = | 2.090, | DISPLACEMENT = | 0.96394212326014D+00 |
| TIME = | 2.100, | DISPLACEMENT = | 0.96858404264596D+00 |
| TIME = | 2.110, | DISPLACEMENT = | 0.97320913182266D+00 |
| TIME = | 2.120, | DISPLACEMENT = | 0.97781987758092D+00 |
| TIME = | 2.130, | DISPLACEMENT = | 0.98242001202845D+00 |
| TIME = | 2.140, | DISPLACEMENT = | 0.98701187638821D+00 |
| TIME = | 2.150, | DISPLACEMENT = | 0.99159818288714D+00 |
| TIME = | 2.160, | DISPLACEMENT = | 0.99618157630830D+00 |
| TIME = | 2.170, | DISPLACEMENT = | 0.10007641987912D+01 |
| TIME = | 2.180, | DISPLACEMENT = | 0.10053494399595D+01 |
| TIME = | 2.190, | DISPLACEMENT = | 0.10099393168333D+01 |
| TIME = | 2.200, | DISPLACEMENT = | 0.10145362237750D+01 |
| TIME = | 2.210, | DISPLACEMENT = | 0.10191420575858D+01 |
| TIME = | 2.220, | DISPLACEMENT = | 0.10237599774067D+01 |
| TIME = | 2.230, | DISPLACEMENT = | 0.10283917712906D+01 |
| TIME = | 2.240, | DISPLACEMENT = | 0.10330396140693D+01 |
| TIME = | 2.250, | DISPLACEMENT = | 0.10377051845379D+01 |

TIME = 2.260, DISPLACEMENT = 0.10423914489334D+01  
TIME = 2.270, DISPLACEMENT = 0.10470999932314D+01  
TIME = 2.280, DISPLACEMENT = 0.10518328029171D+01  
TIME = 2.290, DISPLACEMENT = 0.10565913655695D+01  
TIME = 2.300, DISPLACEMENT = 0.10613784905069D+01  
TIME = 2.310, DISPLACEMENT = 0.11476631005725D+01  
TIME = 2.320, DISPLACEMENT = 0.11533185695767D+01  
TIME = 2.330, DISPLACEMENT = 0.11590500263745D+01  
TIME = 2.340, DISPLACEMENT = 0.11648631801910D+01  
TIME = 2.350, DISPLACEMENT = 0.11707621604698D+01  
TIME = 2.360, DISPLACEMENT = 0.11767516974758D+01  
TIME = 2.370, DISPLACEMENT = 0.11828365811738D+01  
TIME = 2.380, DISPLACEMENT = 0.11890210681560D+01  
TIME = 2.390, DISPLACEMENT = 0.11953112520959D+01  
TIME = 2.400, DISPLACEMENT = 0.12017115269290D+01  
TIME = 2.410, DISPLACEMENT = 0.12082269375580D+01  
TIME = 2.420, DISPLACEMENT = 0.12148619575064D+01  
TIME = 2.430, DISPLACEMENT = 0.12216230328019D+01  
TIME = 2.440, DISPLACEMENT = 0.12285147870224D+01  
TIME = 2.450, DISPLACEMENT = 0.12355425415795D+01  
TIME = 2.460, DISPLACEMENT = 0.12427109991126D+01  
TIME = 2.470, DISPLACEMENT = 0.12500269925297D+01  
TIME = 2.480, DISPLACEMENT = 0.12574953844797D+01  
TIME = 2.490, DISPLACEMENT = 0.12651217887352D+01  
TIME = 2.500, DISPLACEMENT = 0.12729111430751D+01  
TIME = 2.510, DISPLACEMENT = 0.12808706969625D+01  
TIME = 2.520, DISPLACEMENT = 0.12890055553986D+01  
TIME = 2.530, DISPLACEMENT = 0.12973216346622D+01  
TIME = 2.540, DISPLACEMENT = 0.13058241075455D+01  
TIME = 2.550, DISPLACEMENT = 0.13145206649727D+01  
TIME = 2.560, DISPLACEMENT = 0.13234166514682D+01



## Appendix B

Another sample program - use physical units rather nondimensionalized numbers.



```

PROGRAM GPLATE
DOUBLE PRECISION DISPV(1024)
DIMENSION TIME(1024)
INTEGER YORNO

C
      WRITE(1,*)"THIS PROGRAM WILL COMPUTE THE DISPLACEMENT U SUB I AT"
      WRITE(1,*)" A POINT ON THE SURFACE OF AN INFINITE PLATE DUE TO A"
      WRITE(1,*)" POINT STEP IMPULSIVE FORCE F SUB J OR A POINT LINEAR"
      WRITE(1,*)" RAMP FORCE DIPOLE F SUB J,K ACTING AT ANOTHER POINT"
      WRITE(1,*)" ON THE SURFACE OF THE PLATE."
      WRITE(1,*)" INPUTS REQUIRED ARE PROMPTED AND TYPED IN ON THE"
      WRITE(1,*)" KEYBOARD."
      WRITE(1,*)" "
10   WRITE(1,*)" SUBSCRIPTS OF G = ? (i. e. 11,13,131,322, etc.)"
      READ(1,*) INDEX
      WRITE(1,*)" SHEAR WAVE SPEED = ?"
      READ(1,*) SHSPD
      WRITE(1,*)" LONGITUDINAL WAVE SPEED = ?"
      READ(1,*) PSPD
100  WRITE(1,*)"ARE SOURCE AND DETECTER ON THE SAME SIDE OF THE PLATE"
      WRITE(1,*)" PLEASE TYPE 1 FOR YES"
      READ(1,*) YORNO
      IF (YORNO.EQ.1) THEN
          ZD = 0.5
      ELSE
          ZD = -0.5
      END IF
      WRITE(1,*)" WHAT IS THE THICKNESS OF THE PLATE ?"
      READ(1,*) HTHICK
      WRITE(1,*)" WHAT IS THE DISTANCE BETWEEN DETECTER AND SOURCE"
      WRITE(1,*)" OR EPICENTER ?"
      READ(1,*) X
      WRITE(1,*)" SAMPLING TIME INTERVAL = ?"
      READ(1,*) DT
      WRITE(1,*)" NUMBER OF SAMPLING POINTS = ? (MAX = 1024)"
      READ(1,*) NPT
      WRITE(1,*)" SHEAR MODULUS OF THE PLATE = ?"
      READ(1,*) SHMDUL
      WRITE(1,*)" THE OUTPUT WILL BE TIME VERSUS DISPLACEMENT."
      WRITE(1,*)" "
C
      ALPHA = SHSPD/PSPD
      XD = X/HTHICK
      TSCALE = HTHICK/SHSPD
      DELTAT = DT/TSCALE
      DSCALE = 1./(2.*ASIN(1.0)*SHMDUL*HTHICK)

C
      WRITE(2,*)" "
      WRITE(2,*)" THE FOLLOWINGS ARE INPUT PARAMETERS:"
      WRITE(2,*)           INDEX = ',INDEX
      WRITE(2,*)           SHEAR WAVE SPEED = ',SHSPD
      WRITE(2,*)           LONGITUDINAL WAVE SPEED = ',PSPD
      WRITE(2,*)           THICKNESS OF THE PLATE = ',HTHICK
      WRITE(2,*)           SAMPLING TIME INTERVAL = ',DT

```

```
      WRITE(2,*)'          SHEAR MODULUS = ',SHMDUL
      WRITE(2,*)"NUMBER OF POINTS TO BE COMPUTED = ",NPT
      IF (YORNO.EQ.1) THEN
          WRITE(2,*)" THE DETECTER AND THE SOURCE ARE ON THE SAME SIDE"
      ELSE
          WRITE(2,*)"THE DETECTER AND THE SOURCE ARE ON OPPOSITE SIDES"
      END IF
C
      WRITE(2,*) ''
      CALL GREENFCT(ALPHA,XD,ZD,INDEX,DELTAT,NPT,DISPV)
      DO 200 I=1,NPT
          TIME(I)=DELTAT*REAL(I)*TSCALE
          DISPV(I)=DISPV(I)*DSCALE
          WRITE(2,*)" TIME = ",TIME(I), "    DISPLACEMENT = ",DISPV(I)
200 CONTINUE
      STOP
      END
```

## Appendix C

Listing of Subroutines for Computing Green's function of an infinite plate.



```

C*****SUBROUTINE GREENFCT(ALPHA,XD,ZD,INDEX,TDELTA,NPOINT,DISPL)*****
C
C      TO COMPUT GREEN'S FUNCTION - DISPLACEMENT AS A FUNCTION OF TIME
C      - FOR GIVEN TEST CONFIGURATION
C
C      INPUTS:
C          ALPHA = SHEAR WAVE SPEED / LONGITUDINAL WAVE SPEED
C          XD = X COORDINATE OF THE DETECTOR
C          ZD = Z COORDINATE OF DETECTOR
C              ZD = 0.5 => DETECTOR & SOURCE ON THE SAME SIDE
C              ZD = -0.5 => DETECTOR & SOURCE ON OPPOSIT SIDES
C          INDEX = SUBSCRIPT OF GREEN'S FUNCTION, i.e. 33, 322
C          TDELTA = TIME INCREMENT
C          NPOINT = TOTAL NUMBER OF POINTS TO BE COMPUTED
C
C      OUTPUT:
C          DISPL = DISPLACEMENT ** DOUBLE PRECISION, MUST BE
C                  DIMENSIONED (NPOINT)
C-----NOTE 1:
C          ALL INPUTS AND OUTPUTS ARE NONDIMENSIONALIZED PARAMETERS
C          DISPL = NORMALIZED DISPLACEMENT = ACTUAL DISPLACEMENT * PI
C                  * SHEAR MODULUS * PLATE THICKNESS / FORCE
C          T = NORMALIZED TIME = ACTUAL TIME *SHEAR WAVE SPEED / PLATE
C                  THICKNESS
C          DISPL MUST BE DIMENSIONED IN THE CALLING PROGRAM (NPOINT)
C
C-----NOTE 2:
C          THE ARRIVAL TIMES ARE STORED IN RAYTIME(N,I)
C          N = THE Nth RAY
C          RAYTIME(N,3) = THE ARRIVAL TIME
C          RAYTIME(N,1) = N1, NUMBER OF P TRIPS
C          RAYTIME(N,2) = N2, NUMBER OF S TRIPS
C          IF N2<0, IT IS A HEAD WAVE
C          IF N1=-1 , IT IS SURFACE WAVE
C          IF N1=0 AND N2=0, IT IS A RAYLEIGH WAVE
C
C          THE TIME OF ARRIVAL IMFORMATION IS ALSO PRINTED ON LU 6
C
C-----DOUBLE PRECISION DISPL(NPOINT)
C      DIMENSION TA(101),CN(101,3),RAYTIME(101,3)
C      NRAY = NUMBER OF MAXIMUM RAYS EXPECTED
C      DATA NRAY/101/
C
C      CALL INIT(ALPHA,XD,ZD,INDEX)

```

```

CALL TIMEARRI(ALPHA,XD,ZD,NRAY,RAYTIME,TA,CN)          GR000550
WRITE(6,*) '      I      N1      N2      TIME ARRIVAL'   GR000560
DO 500 I=1,NRAY                                       GR000570
      WRITE (6,499) I,RAYTIME(I,1),RAYTIME(I,2),RAYTIME(I,3)
499     FORMAT(2X,15,3X,3F10.5)                         GR000580
500 CONTINUE                                         GR000590
      DO 1000 I=1,NPOINT                                GR000600
         TIMEI = FLOAT(I)*TDELTA                      GR000610
         CALL GREENSUB(ALPHA,INDEX,XD,ZD,NRAY,TIMEI,RAYTIME,DISPL(I))
         WRITE(6,*)'TIME = ',TIMEI,'           DISPL = ',DISPL(I)
1000 CONTINUE                                         GR000620
      RETURN                                            GR000630
      END                                              GR000640
C*****
C
      SUBROUTINE INIT(ALPHA,XD,ZD,INDEX)                GR000650
C
C*****
IMPLICIT DOUBLE COMPLEX (Y)                           GR000660
IMPLICIT DOUBLE PRECISION (D)                         GR000670
COMMON /BLK0/DXD,DT,DZ,DA                           GR000680
COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA             GR000690
COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO GR000700
COMMON /BLK5/YR,Y1,Y2                               GR000710
DXD = DBLE(XD)                                      GR000720
DZ  = DBLE(ZD)                                      GR000730
DA  = DBLE(ALPHA)                                    GR000740
IF (ZD.EQ.0.5) KASE = 1                            GR000750
IF (ZD.EQ.-0.5) KASE = 2                           GR000760
IF (INDEX.LT.100) THEN                           GR000770
  K = 0
  M = INDEX/10                                     GR000780
  L = INDEX - M*10
ELSE IF ((INDEX.GT.100).AND.(INDEX.LT.334)) THEN GR000790
  M = INDEX/100
  L = (INDEX - M*100)/10
  K = (INDEX - M*100 - L*10)
ELSE
  WRITE(6,*) '  WRONG INDEX  '
  PAUSE
END IF
YI = DCMPLX(0.0D0,1.0D0)                           GR000800
YONE = DCMPLX(1.0D0,0.0D0)                          GR000810
YTWO = DCMPLX(2.0D0,0.0D0)                          GR000820
YTHREE = DCMPLX(3.0D0,0.0D0)                         GR000830
YFOUR = DCMPLX(4.0D0,0.0D0)                         GR000840
YEIGHT = DCMPLX(8.0D0,0.0D0)                         GR000850
YZERO = DCMPLX(0.0D0,0.0D0)                          GR000860
CALL RAYRT(YR,Y1,Y2,ALPHA)                          GR000870
RETURN
END
C*****
C
      SUBROUTINE GREENSUB(ALPHA,INDEX,XD,ZD,NRAY,T,RAYTIME,DISPL) GR000880
C

```

```

***** SUBROUTINE TO COMPUTE THE GREENS FUNCTION OF A PLATE AT T ***** GR001100
C      SUBROUTINE TO COMPUTE THE GREENS FUNCTION OF A PLATE AT T          GR001110
C                                                               GR001120
C
C   INPUTS:                                                       GR001130
C
C     CONFIGURATION PARAMETERS:                                         GR001140
C
C       DETECTOR LOCATION:                                              GR001150
C
C         X = XD                                                       GR001160
C
C         Y = 0                                                       GR001170
C
C         Z = ZD, ZD=0.5 => TOP; ZD=-0.5 => BOTTOM                 GR001180
C
C     SOURCE LOCATION:                                                 GR001190
C
C       X0 = 0                                                       GR001200
C
C       Y0 = 0                                                       GR001210
C
C       Z0 = +0.5 (ON TOP OF THE PLATE)                                GR001220
C
C                                                               GR001230
C
C   MATERIALS PARAMETERS:                                             GR001240
C
C     ALPHA = RATIO OF SHEAR VELOCITY TO LONGITUDINAL VELOCITY        GR001250
C
C                                                               GR001260
C
C   OTHER PARAMETERS:                                                 GR001270
C
C     INDEX: SUBSCRIPTS OF GREENS FUNCTION i.e. 33, or 113            GR001280
C
C     NRAY: MAXIMUM NUMBER OF RAYS EXPECTED. ~100                      GR001290
C
C     RAYTIME: TIME OF ARRIVAL TABLE OF DIMENSION (NRAY,3)             GR001300
C
C                                                               GR001310
C
C     T = NORMALIZED TIME = ACTUAL TIME *SHEAR WAVE SPEED / PLATE    GR001320
C
C               THICKNESS                                              GR001330
C
C                                                               GR001340
C
C     DISPLACEMENT WILL BE CORRECT ONLY IF T < RAYTIME(NRAY,3)          GR001350
C
C                                                               GR001360
C
C   OUTPUT:                                                       GR001370
C
C     DISPL = NORMALIZED DISPLACEMENT = ACTUAL DISPLACEMENT * PI        GR001380
C
C               * SHEAR MODULUS * PLATE THICKNESS / FORCE                GR001390
C
C                                                               GR001400
C
C     NOTE: SUBROUTINE INIT(ALPHA,XD,ZD,INDEX) AND                      GR001410
C
C           SUBROUTINE TIMEARRI(.....) SHOULD BE CALLED FIRST IN          GR001420
C
C           THE MAIN PROGRAM.                                            GR001430
C
C                                                               GR001440
C
C     IMPLICIT DOUBLE COMPLEX (Y)                                         GR001450
C
C     IMPLICIT DOUBLE PRECISION (D)                                       GR001460
C
C     DOUBLE PRECISION DINTG                                           GR001470
C
C     DIMENSION RAYTIME(NRAY,3)                                         GR001480
C
C     COMMON /BLK0/DXD,DT,DZ,DA                                         GR001490
C
C     COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA                           GR001500
C
C     COMMON /BLK2/YD,Y,YP,YQ,YR0,YRP,YRM,YDELTA,YPSQP1,YSQP1,        GR001510
C
C     1   YPSQM1,YQSQM1,YSQR1,YSQRQ1,YY,YA,YAA,YETA1,YETA2          GR001520
C
C     2   ,YDPHI                                               GR001530
C
C     COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO              GR001540
C
C     COMMON /BLK5/YR,Y1,Y2                                              GR001550
C
C                                                               GR001560
C
C     EXTERNAL DINTG                                              GR001570
C
C                                                               GR001580
C
C                                                               GR001590
C
C     IF ( (INDEX.EQ.12).OR.(INDEX.EQ.21).OR.                         GR001600
C
C     +   (INDEX.EQ.23).OR.(INDEX.EQ.32).OR.                          GR001610
C
C     +   (INDEX.EQ.121).OR.(INDEX.EQ.211).OR.                         GR001620
C
C     +   (INDEX.EQ.231).OR.(INDEX.EQ.321).OR.                         GR001630
C
C     +   (INDEX.EQ.123).OR.(INDEX.EQ.213).OR.                         GR001640

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```

+      (INDEX.EQ.233).OR.(INDEX.EQ.323).OR.          GR001650
+      (INDEX.EQ.132).OR.(INDEX.EQ.312).OR.          GR001660
+      (INDEX.EQ.112).OR.(INDEX.EQ.222).OR.(INDEX.EQ.332) ) THEN   GR001670
    DISPL=0.0D0                                     GR001680
    RETURN                                         GR001690
  ELSE                                              GR001700
C                                                 GR001710
    DZERO=0.D0                                      GR001720
C     NMAX = NRAY; Maximum number of rays. The same as DIM RAYTIME   GR001730
  NMAX = NRAY                                       GR001740
  IF(T.GT.(RAYTIME(NRAY,3))) THEN                 GR001750
    WRITE(6,*)'*****'
    WRITE(6,*)'ERROR IN SUBROUTINE GREEN'
    WRITE(6,*)'T > TARRIVAL(NRAY), INCREASE NRAY.'
    WRITE(6,*)'PROGRAM IS PAUSED'
    WRITE(6,*)'*****'
    PAUSE 'GREENSUB'
  ENDIF                                             GR001800
C                                                 GR001810
  DT=DBLE(T)                                       GR001820
C                                                 GR001830
C                                                 GR001840
C                                                 GR001850
C                                                 GR001860
  DSURF = 0.D0                                      GR001870
  IF (KASE.EQ.1) THEN                               GR001880
    CALL SFWAVE(ALPHA,XD,T,INDEX,DSURF)
  ELSE IF (KASE.EQ.2)THEN                           GR001890
    DSURF = 0.D0                                      GR001900
  END IF                                             GR001910
  DISPL=0.D0                                         GR001920
C                                                 GR001930
  DO 8000 N=1,NMAX
    IF (T.LE.RAYTIME(N,3)) GO TO 8001
    N1 = RAYTIME(N,1)
    N2 = RAYTIME(N,2)
C                                                 GR001940
  IF((N1.EQ.0).AND.(N2.EQ.0)) GOTO 8000          GR001950
  IF((N1.LT.0).AND.(N2.EQ.0)) GOTO 8000          GR001960
C*** IT IS RAYLEIGH WAVE ARRIVAL WHICH HAS BEEN TAKEN   GR001970
C CARE OF BY SUBROUTINE SFWAVE                      GR001980
  IF(N2.LT.0) THEN                                 GR001990
    IT IS A HEAD WAVE                            GR002000
    N2=-N2                                         GR002010
    DK2=DBLE(REAL(N2))
    DTSARR = DSQRT(DXD*DxD + DK2*DK2)           GR002020
    IF (DT.GE.DTSARR) GO TO 8000
    DHEAD = (DT/DA)-DK2*DSQRT(1.0D0-DA*DA)/DA
    DRAYI=DINTG(-DXD,-DHEAD)
  ELSE                                              GR002040
    IT IS A REGULAR RAY                         GR002050
    DRAYI=DINTG(DZERO,-DXD)
  END IF                                           GR002060
C                                                 GR002070
  DISPL = DISPL + DRAYI                          GR002080
C8000  Y=YZERO ;set try root eq. 0             GR002090
  8000  Y=YZERO                                     GR002100

```

```

8001  DISPL=DISPL/(6.28318530717959D0)          GR002200
      DISPL=DISPL+DSURF                         GR002210
      RETURN                                         GR002220
      END IF                                         GR002230
      END                                           GR002240
*****
C
C      DOUBLE PRECISION FUNCTION DINTG(DINIT,DEND)    GR002270
C
C*****
C
C      IMPLICIT DOUBLE PRECISION (D)                  GR002310
C
C
C      DOUBLE PRECISION EPS,R,E,W(50,6),FMIN,FMAX,F   GR002340
C      INTEGER NINT,NMAX,KF,IFLAG                      GR002350
C      LOGICAL RST                                       GR002360
C
C
C      EXTERNAL F                                       GR002390
C
C
C      EPS = 1.D-6                                     GR002410
C      RST = .FALSE.                                  GR002420
C      NMAX = 50                                      GR002430
C          NINT = 1                                    GR002440
C          W(1,1) = DINIT                           GR002450
C          W(2,1) = DEND                            GR002460
C
C      CALL DGLQ1(F,DINIT,DEND,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX,KF
C      1 ,IFLAG)                                     GR002480
C
C
C      DINTG=R                                         GR002500
C      RETURN                                         GR002510
C      END                                            GR002520
C*****
C
C      DOUBLE PRECISION FUNCTION F(DS)                GR002570
C
C*****
C
C      IMPLICIT DOUBLE COMPLEX (Y)                   GR002610
C      IMPLICIT DOUBLE PRECISION (D)                 GR002620
C      DOUBLE PRECISION ETA1,ETA2SQ,DD,AA           GR002630
C      DIMENSION YQD(3,3),YRSTAR(3,3),YQS(3,3)     GR002640
C      COMMON /BLK0/DXD,DT,DZ,DA                     GR002650
C      COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA       GR002660
C      COMMON /BLK2/YD,Y,YP,YQ,YR0,YRP,YRM,YDELTA,YSQP1,YSQP1,
C      1      YPSQM1,YSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2   GR002670
C      2      ,YDPHI                                GR002680
C      COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO   GR002690
C
C      DK1=DBLE(REAL(N1))                          GR002700
C      DK2=DBLE(REAL(N2))                          GR002710
C      D=DS                                         GR002720

```

```

C                               GR002750
CALL PHIEQ0(DK1,DK2,DA,DT,DXD,DS,Y,DERR,NROOT)      GR002760
C                               GR002770
WRITE(6,*)DK1,DK2,DS,Y,DERR,NROOT
IF (NROOT.EQ.0) THEN
  F=0.D0
C                               GR002790
C                               GR002800
ELSE
CALL COMMON(DS)
CALL RSTAR(YRSTAR,IDUMMY)
CALL QD(YQD, IDUMMY)
CALL QSU(YQS, IDUMMY)
C                               GR002860
YINTG = YZERO
DO 200 I=1,2
  DO 100 J=1,2
    YINTG=YINTG+YQD(M,I)*YRSTAR(I,J)*YQS(L,J)
100   CONTINUE
200   CONTINUE
C                               GR002940
IF ((N1.EQ.0).AND.(N2.GT.0))
1   YINTG=YINTG+YQD(M,3)*YQS(L,3)
C                               GR002950
C                               GR002960
YINTG=YINTG*YDphi/CDSQRT(DCMPLX(DXD*DxD-DS*DS))
F=DBLE(YINTG)
C                               GR002990
C                               GR003000
END IF
RETURN
END
*****
C***** SUBROUTINE PHIEQ0(DK1,DK2,DA,DT,D,YROOT,DERR,NROOT)      GR003030
C                               GR003040
C***** GR003050
C                               GR003060
C***** GR003070
C                               GR003080
C      GIVEN DK1,DK2,DA,DT,D TO SOLVE PHI(YROOT)=0      GR003090
IMPLICIT DOUBLE PRECISION (D)
IMPLICIT DOUBLE COMPLEX (Y)
DOUBLE PRECISION T,R,A,C1,C2
COMMON /PARM1/T,R,A,C1,C2
C                               GR003140
YOLD=YROOT
IER=0
DYREAL=0.D0
YDphi=DCMPLX(0.D0,0.D0)
T = DT
R = D
A = DA
C1 = DK1
C2 = DK2
EPS=1.E-12
IEND=100
C                               GR003260
NROOT=1
IF (DK2.EQ.0.D0) THEN
C                               GR003270
C                               GR003280
C                               GR003290

```

```

DSTSQ = (DT/DA)*(DT/DA)-DK1*DK1           GR003300
DSTAR = DSQRT(DSTSQ)                      GR003310
IF (D.LT.0.D0)DSTAR=-DSTAR                GR003320
C
IF (D.EQ.0.D0) THEN                         GR003330
DYIMAG=-DK1/(DSTAR*DA)                     GR003340
YROOT = DCMPLX(0.D0,DYIMAG)                 GR003350
ELSE IF (D.EQ.DSTAR) THEN                  GR003360
DYREAL=DT/(DA*DA*DSTAR)                   GR003370
YROOT = DCMPLX(DYREAL,0.0D0)                 GR003380
ELSE                                         GR003390
DYREAL = D*DT/(DA*DA*DSTSQ)                 GR003400
DSMD=DSTSQ-D*D                           GR003410
IF(DSMD.LT.0.D0) WRITE(6,*) DT,D,DSMD,DSTSQ   GR003420
DYIMAG = -(DK1/(DSTSQ*DA))*DSQRT(DSMD)      GR003430
YROOT = DCMPLX(DYREAL,DYIMAG)                 GR003440
END IF                                       GR003450
YOLD = YROOT                                  GR003460
CALL DCRTNI(YROOT,YPHI,YDPHI,YOLD,EPS,IEND,IER)  GR003470
GR003480
C
ELSE IF (DK1.EQ.0.D0) THEN                  GR003490
DTHARR = DA*DABS(DXD)+DK2*DSQRT(1.0D0-DA*DA)  GR003510
DTSARR = DSQRT(DXD*DXD+DK2*DK2)             GR003520
DSTSQ = (DT)*(DT)-DK2*DK2                   GR003530
DSTAR = DSQRT(DSTSQ)                        GR003540
GR003550
C
IF (DT.GT.DTSARR) THEN                      GR003560
IF (D.LT.0.D0) DSTAR=-DSTAR                GR003570
C
IF (D.EQ.0.D0) THEN                         GR003580
DYIMAG=-DK2/DABS(DSTAR)                    GR003590
YROOT = DCMPLX(0.D0,DYIMAG)                 GR003600
ELSE IF (D.EQ.DSTAR) THEN                  GR003610
DYREAL=DT/(DSTAR)                          GR003620
YROOT = DCMPLX(DYREAL,0.0D0)                 GR003630
ELSE                                         GR003640
DYREAL = D*DT/(DSTSQ)                      GR003650
DYIMAG = -(DK2/DSTSQ)*DSQRT(DSTSQ-D*D)    GR003660
YROOT = DCMPLX(DYREAL,DYIMAG)                 GR003670
END IF                                       GR003680
GR003690
ELSE IF ((DT.LT.DTSARR).AND.(DT.GT.DTHARR)) THEN  GR003700
DHEAD = (DT/DA)-DK2*DSQRT(1.0D0-DA*DA)/DA   GR003710
IF (D.LT.0.D0)DHEAD=-DHEAD                  GR003720
IF (D.LT.0.D0)DSTAR=-DSTAR                  GR003730
C
IF (D.EQ.0.D0) THEN                         GR003740
DYIMAG=-DK2/DSTAR                          GR003750
YROOT = DCMPLX(0.D0,DYIMAG)                 GR003760
ELSE IF (D.EQ.DSTAR) THEN                  GR003770
DYREAL=DT/(DSTAR)                          GR003780
YROOT = DCMPLX(DYREAL,0.0D0)                 GR003790
ELSE IF (DABS(D).GT.DABS(DSTAR)) THEN     GR003800
IF (D.LT.0.D0) THEN                         GR003810
DYREAL = (D*DT-DK2*DSQRT(D*D-DSTSQ))/(DSTSQ)  GR003820
ELSE IF (D.GT.0.D0) THEN                  GR003830
GR003840

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        DYREAL = (D*DT+DK2*DSQRT(D*D-DSTSQ))/(DSTSQ)           GR003850
        ENDIF
        DYIMAG = 0.D0                                         GR003870
        YROOT = DCMPLX(DYREAL,DYIMAG)                         GR003880
    END IF                                                 GR003890
    END IF                                                 GR003900
    YOLD = YROOT                                         GR003910
    CALL DCRTNI(YROOT,YPHI,YDPHI,YOLD,EPS,IEND,IER)       GR003920
C
C ELSE IF ((DK1.NE.0.D0).AND.(DK2.NE.0.D0)) THEN          GR003930
    IF((D.EQ.0.D0).OR.(YOLD.EQ.DCMPLX(0.D0,0.D0))) THEN   GR003950
        CALL DEQO(DT,DA,DK1,DK2,YROOT,NOROOT)               GR003960
        ENDIF
        YOLD = YROOT                                         GR003970
        CALL DCRTNI(YROOT,YPHI,YDPHI,YOLD,EPS,IEND,IER)       GR003990
    END IF                                                 GR004000
    IF(IER.NE.0) NOROOT=0                                 GR004010
C
C TEST ROOT TO FIND ERROR                            GR004020
C
C DERR=CDABS(YPHI)                                  GR004040
C
C RETURN                                              GR004060
C
C*****SUBROUTINE DEQO(DT,DA,DK1,DK2,Y,NOROOT)          GR004090
C
C*****COMPUTE THE ROOT BY SOLVING QUARTIC EQ.          GR004110
C
C IMPLICIT DOUBLE PRECISION (D)                      GR004120
C IMPLICIT DOUBLE COMPLEX (Y)                        GR004130
C DOUBLE PRECISION B                                GR004140
C DIMENSION DB(5),B(3),DROOT1(2),DROOT2(2)          GR004150
C
C
C YF(Y)=-DCMPLX(DT,0.D0)*Y-DCMPLX(DK1,0.D0)*          GR004160
C 1      CDSQRT(Y*Y*DCMPLX(DA*DA,0.D0)-DCMPLX(1.D0,0.D0))  GR004170
C 2      -DCMPLX(DK2,0.D0)*CDSQRT(Y*Y-DCMPLX(1.D0,0.D0))  GR004180
C DT2 = DT*DT                                         GR004190
C DK12 = DK1*DK1                                       GR004200
C DK22 = DK2*DK2                                       GR004210
C DAA=DA*DA                                         GR004220
C
C DB(1)=((DT2-DK12*DAA-DK22)**2 - 4.D0*DK12*DK22*DAA)  GR004230
C DB(3)=2.D0*((DT2-DK12*DAA-DK22)*(DK12+DK22)           GR004240
C 1      +2.D0*DK12*DK22*(DAA+1.D0))                   GR004250
C DB(5)=(DK12+DK22)**2-4.D0*DK12*DK22                  GR004260
C
C B(1) = DB(1)                                         GR004270
C B(2) = DB(3)                                         GR004280
C B(3) = DB(5)                                         GR004290
C TOL=1.E-20                                         GR004300
C
C CALL QUADRTIC EQ. TO FIND Y**2                     GR004310

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CALL QUADRT(B,DROOT1,DROOT2,TOL,NOROOT) GR004400
C GR004410
Y1=CDSQRT(DCMPLX(DROOT1(1),DROOT1(2))) GR004420
Y2=CDSQRT(DCMPLX(DROOT2(1),DROOT2(2))) GR004430
IF (DIMAG(Y1).GT.0.00) Y1=-Y1 GR004440
IF (DIMAG(Y2).GT.0.00) Y2=-Y2 GR004450
YF1=YF(Y1) GR004460
YF2=YF(Y2) GR004470
DERR1=CDABS(YF(Y1)) GR004480
DERR2=CDABS(YF(Y2)) GR004490
IF (DERR1.LT.DERR2) Y=Y1 GR004500
IF (DERR1.GE.DERR2) Y=Y2 GR004510
C GR004520
IF (DIMAG(Y).GE.0.00) Y=DCONJG(Y) GR004530
C GR004540
RETURN GR004550
END GR004560
*****
C***** DOUBLE COMPLEX FUNCTION PHIO(Z) GR004570
C GR004580
C***** GR004590
C GR004600
C***** GR004610
C GR004620
C FUNCTION PHIO GR004630
C GR004640
DOUBLE PRECISION T,R,A,C1,C2 GR004650
DOUBLE COMPLEX Z,YAA,YONE GR004660
COMMON /PARM1/T,R,A,C1,C2 GR004670
C GR004680
YONE=DCMPLX(1.00,0.00) GR004690
YAA=DCMPLX(A*A,0.00) GR004700
C GR004710
PHIO=DCMPLX(R,0.00)-DCMPLX(T,0.00)*Z GR004720
1 -DCMPLX(C1,0.00)*CDSQRT(YAA*Z*Z-YONE) GR004730
2 -DCMPLX(C2,0.00)*CDSQRT(Z*Z-YONE) GR004740
C GR004750
RETURN GR004760
END GR004770
*****
C***** DOUBLE COMPLEX FUNCTION DPHIO(Z) GR004780
C GR004790
C***** GR004800
C GR004810
C***** GR004820
C GR004830
C FUNCTION DPHIO GR004840
C GR004850
C GR004860
DOUBLE PRECISION T,R,A,C1,C2 GR004870
DOUBLE COMPLEX Z,YAA,YONE GR004880
COMMON /PARM1/T,R,A,C1,C2 GR004890
C GR004900
YONE=DCMPLX(1.00,0.00) GR004910
YAA=DCMPLX(A*A,0.00) GR004920
C GR004930
DPHIO=-DCMPLX(T,0.00) GR004940

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1   -DCMPLX(C1,0.D0)*YAA*Z/CDSQRT(YAA*Z*Z-YONE)           GR004950
2   -DCMPLX(C2,0.D0)*Z/CDSQRT(Z*Z-YONE)                   GR004960
C
C      RETURN                                              GR004970
C      END                                                 GR004980
C*****SUBROUTINE COMMON(DS)                                GR005000
C
C      SUBROUTINE COMMON(DS)                                GR005010
C
C      TO COMPUTE PARAMETERS DEPENDING ON Y AND TO PUT IN    GR005020
C      A COMMON BLK2                                         GR005030
C
C      IMPLICIT DOUBLE COMPLEX (Y)                           GR005040
C      IMPLICIT DOUBLE PRECISION (D)                         GR005050
C      DOUBLE PRECISION ETA1,ETA2SQ,DD,AA                  GR005060
C      COMMON /BLK0/DXD,DT,DZ,DA                          GR005070
C      COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA            GR005080
C      COMMON /BLK2/YD,Y,YP,YQ,YR0,YRP,YRM,YDELTA,YPSQP1,YQSQP1,   GR005090
1      YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2   GR005100
2      ,YDPHI                                           GR005110
C      COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO   GR005120
C
C      AA=DA*DA                                         GR005130
C      DD=DS                                           GR005140
C      D=DS                                           GR005150
C      DN1=DBLE(REAL(N1))                            GR005160
C      DN2=DBLE(REAL(N2))                            GR005170
C      YZERO=DCMPLX(0.D0,0.D0)                         GR005180
C      YI=DCMPLX(0.D0,1.D0)                           GR005190
C      YAA=DCMPLX(AA,0.D0)                            GR005200
C      YA=DCMPLX(DA,0.D0)                            GR005210
C      YONE=DCMPLX(1.0D0,0.0D0)                         GR005220
C      YTWO=DCMPLX(2.D0,0.D0)                           GR005230
C      YFOUR=DCMPLX(4.0D0,0.D0)                         GR005240
C
C      ETA1=DS/DXD                                     GR005250
C      ETA2SQ=1.D0-ETA1*ETA1                         GR005260
C      YETA1 = DCMPLX(ETA1,0.D0)                      GR005270
C      YETA2S= DCMPLX(ETA2SQ,0.D0)                    GR005280
C      YETA2=CDSQRT(YETA2S)                           GR005290
C
C      YD = DCMPLX(DD,0.D0)                           GR005300
C      YY=Y*Y                                         GR005310
C      YP=CDSQRT(YAA*YY-YONE)                         GR005320
C      YQ=CDSQRT(YY-YONE)                           GR005330
C      YPSQP1=YAA*YY                                 GR005340
C      YQSQP1=YY                                    GR005350
C      YQSQM1=YQ*YQ-YONE                           GR005360
C      YPSQM1=YP*YP - YONE                          GR005370
C
C      YSQRP1=Y*YA                                  GR005380
C      YSQRQ1=Y                               GR005390
C      YDELTA=YQSQM1*YQSQM1+YFOUR*YP*YQ          GR005400

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YM=(YQSQM1*YQSQM1-YFOUR*YP*YQ) GR005500
YR0=YM/YDELTA GR005510
YRP=-(YFOUR*YP*YQSQM1)/(YA*YDELTA) GR005520
YRM=(YR0*YR0-YONE)/YRP GR005530
C GR005540
YDPHI=YONE/(YD+(DCMPLX(DN1,0.D0)/YP)+(DCMPLX(DN2,0.D0)/YQ)) GR005550
RETURN GR005560
END GR005570
C***** GR005580
C GR005590
C SUBROUTINE RSTAR(YRSTAR, IDUMMY) GR005600
C GR005610
C***** GR005620
C GR005630
C TO COMPUTE RSTAR (JS,J2) EQ.6.39 GR005640
C GR005650
C IMPLICIT DOUBLE COMPLEX (Y) GR005660
C IMPLICIT DOUBLE PRECISION (D) GR005670
C DIMENSION YRSTAR(3,3),KSUM(20) GR005680
C COMMON /BLK0/DXD,DT,DZ,DA GR005690
C COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA GR005700
C COMMON /BLK2/YD,Y,YP,YQ,YR0,YRP,YRM,YDELTA,YPSQP1,YQSQP1, GR005710
1 YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2 GR005720
2 ,YDPHI GR005730
C COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO GR005740
C GR005750
C EXTERNAL KDELTA GR005760
C GR005770
C GR005780
C IDUMMY=0 GR005790
C YRSTAR(3,3) = YONE GR005800
C YRSTAR(1,3) = YZERO GR005810
C YRSTAR(2,3) = YZERO GR005820
C YRSTAR(3,1) = YZERO GR005830
C YRSTAR(3,2) = YZERO GR005840
C GR005850
DO 2000 JS=1,2 GR005860
DO 1000 J2=1,2 GR005870
IF (J2.EQ.1) N1=N1-1 GR005880
IF (J2.EQ.2) N2=N2-1 GR005890
ISIGN = (-1)**(J2*(N1+N2)+N2) GR005900
CALL COEFF(N1,N2,JS,J2,KD,KU,KL,KSUM) GR005910
C WRITE(7,*) N1,N2,JS,J2,KD,KU,KL,KSUM GR005920
YSUM=YZERO GR005930
DO 100 NM=KL,KU GR005940
YSUM=YSUM+DCMPLX(DBLE(REAL(KSUM(NM+1))),0.D0)* GR005950
1 ((YR0*YR0-YONE)/(YR0*YR0))**NM GR005960
100 CONTINUE GR005970
KD2=KDELTA(1,JS)-KDELTA(1,J2) GR005980
C GR005990
YSUM = ( (YR0/YRP)**(KD2) )*YSUM GR006000
RSIGN=REAL(ISIGN) GR006010
DRSIGN=RSIGN GR006020
DKD=DBLE(REAL(KD)) GR006030
YRSTAR(JS,J2)=DCMPLX(DRSIGN,0.D0)*(YR0**((N1+N2))) GR006040

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1 *(DCMPLX(DKD,0.0D)+YSUM) GR006050
IF (J2.EQ.1) N1=N1+1 GR006060
IF (J2.EQ.2) N2=N2+1 GR006070
C GR006080
1000 CONTINUE GR006090
2000 CONTINUE GR006100
RETURN GR006110
END GR006120
C***** GR006130
C GR006140
SUBROUTINE QD(YQD, IDUMMY) GR006150
C GR006160
C***** GR006170
C GR006180
C TO COMPUTE QD(I,J) BASED ON EQ. 6.21 GR006190
C GR006200
IMPLICIT DOUBLE COMPLEX (Y) GR006210
IMPLICIT DOUBLE PRECISION (D) GR006220
COMMON /BLK0/DXD,DT,DZ,DA GR006230
COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA GR006240
COMMON /BLK2/YD,YP,YQ,YR0,YRP,YRM,YDELTA,YPSCP1,YQSCP1,
1 YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2 GR006250
2 ,YDPHI GR006260
COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO GR006270
C GR006280
DIMENSION YQD(3,3) GR006290
C GR006300
IDUMMY=0 GR006310
DO 100 I=1,3 GR006320
   DO 90 J=1,3 GR006330
      YQD(I,J) = YZERO GR006340
90   CONTINUE GR006350
100 CONTINUE GR006360
C GR006370
IF(M.EQ.1) THEN GR006380
   YQD(1,1)=YETA1*YFOUR*YP*YQ*YQSCP1 GR006390
   1 /(YSQRP1*YDELTA) GR006400
   YQD(1,2)=-YTWO*YETA1*YQ*YPSQM1*YSQRQ1 GR006410
   1 /(YDELTA) GR006420
   YQD(1,3) =- YTWO*YETA2 GR006430
C GR006440
ELSE IF (M.EQ.2)THEN GR006450
   YQD(2,1)=YETA2*YFOUR*YP*YQ*YQSCP1 GR006460
   1 /(YSQRP1*YDELTA) GR006470
   YQD(2,2)=-YTWO*YETA2*YQ*YQSQM1*YSQRQ1 GR006480
   1 /(YDELTA) GR006490
   YQD(2,3)=YTWO*YETA1 GR006500
C GR006510
ELSE IF (M.EQ.3) THEN GR006520
   YQD(3,1)=-YTWO*YP*YPSQM1*YQSCP1 GR006530
   1 /(YSQRP1*YDELTA) GR006540
   YQD(3,2)=-YFOUR*YP*YQ*YSQRQ1 GR006550
   1 /(YDELTA) GR006560
   YQD(3,3)=YZERO GR006570
C GR006580

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END IF                                     GR006600
RETURN                                      GR006610
END                                         GR006620
C*****                                         GR006630
C                                         GR006640
C INTEGER FUNCTION KDELTA(I,J)              GR006650
C*****                                         GR006660
C                                         GR006670
C                                         GR006680
C
IF (I.EQ.J) THEN                           GR006690
  KDELTA=1                                 GR006700
ELSE                                         GR006710
  KDELTA=0                                 GR006720
END IF                                       GR006730
RETURN                                      GR006740
END                                         GR006750
C*****                                         GR006760
C                                         GR006770
C INTEGER FUNCTION IBINO(M,N)               GR006780
C*****                                         GR006790
C                                         GR006800
C                                         GR006810
C
IF (M.EQ.N)GO TO 11                         GR006820
IF ((M.LT.0).OR.(N.LT.0)) GO TO 10          GR006830
K=M-N                                       GR006840
IF (K) 10,11,12                            GR006850
10   IBINO = 0                               GR006860
RETURN                                      GR006870
11   IBINO = 1                               GR006880
RETURN                                      GR006890
12   IF (N.EQ.0)GO TO 11                      GR006900
IN = 1                                       GR006910
IP = M                                       GR006920
IQ = 1                                       GR006930
DO 20 I=1,K                                GR006940
  IN = IN*IP/IQ                             GR006950
  IP = IP-1                                GR006960
  IQ = IQ +1                               GR006970
20   CONTINUE                                 GR006980
    IBINO = IN                             GR006990
RETURN                                      GR007000
END                                         GR007010
C*****                                         GR007020
C                                         GR007030
C INTEGER FUNCTION KD1(N1,N2,JS,J2)          GR007040
C*****                                         GR007050
C                                         GR007060
C                                         GR007070
C
EXTERNAL KDELTA                            GR007080
KD1=KDELTA(0,N1)*KDELTA(2,J2)+KDELTA(0,N2)*KDELTA(1,J2)  GR007090
KD1=KD1*KDELTA(JS,J2)                     GR007100
RETURN                                      GR007110
END                                         GR007120
C*****                                         GR007130
C                                         GR007140

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        INTEGER FUNCTION KUPPER(N1,N2,JS,J2)                      GR007150
C                                               GR007160
C*****                                         GR007170
C                                               GR007180
C
        EXTERNAL KDELTA                                     GR007190
        K=N2+KDELTA(2,J2)*KDELTA(1,JS)-KDELTA(1,J2)*KDELTA(2,JS)   GR007200
        IF (N1.LE.K) THEN                                     GR007210
          KUPPER=N1                                         GR007220
        ELSE                                              GR007230
          KUPPER=K                                         GR007240
        END IF                                            GR007250
        RETURN                                           GR007260
        END                                              GR007270
C*****                                         GR007280
C                                               GR007290
C
        INTEGER FUNCTION KLOWER(N1,N2,JS,J2)                  GR007300
C                                               GR007310
C*****                                         GR007320
C                                               GR007330
C
        EXTERNAL KDELTA                                     GR007340
        KLOWER=KDELTA(2,J2)+KDELTA(1,J2)*KDELTA(1,JS)      GR007350
        RETURN                                           GR007360
        END                                              GR007370
C*****                                         GR007380
C                                               GR007390
C
        SUBROUTINE COEFF(N1,N2,JS,J2,KD,KU,KL,KSUM)       GR007400
C                                               GR007410
C*****                                         GR007420
C                                               GR007430
C
C     TO COMPUTE THE COEFFICIENTS FOR THE RSTAR TURM      GR007440
C                                               GR007450
C
C     INPUTS:                                              GR007460
C
C       N1 = NUMBER OF P TRIPS                            GR007470
C
C       N2 = NUMBER OF S TRIPS                            GR007480
C
C       JS = FINAL TRIP TYPE, 1 OR 2                     GR007490
C
C       J2 = INITIAL TRIP TYPE, 1 OR 2                   GR007500
C
C
C     OUTPUT:                                              GR007510
C
C       KD = FIRST COEFF                                GR007520
C
C       KU = UPPER LIMIT IN SUMMATION                  GR007530
C
C       KL = LOWER LIMIT IN SUMMATION                  GR007540
C
C       KSUM = COEFF. IN SUMMATION, MUST BE DIMENSIONED IN CALLING GR007550
C
C           PROGRAM TO BE MAX(N1,N2)                    GR007560
C
C
C-----                                         GR007570
C                                               GR007580
C-----                                         GR007590
C                                               GR007600
C
C     PROGRAM TESTCOEF                                 GR007610
C
C     DIMENSION KSUM(20)                               GR007620
CC***                                         GR007630
C
C     1  WRITE(5,10)                                    GR007640
C
C     10 FORMAT (2X,'TYPE K1,K2,JS,J2')               GR007650
C
C     READ(5,*)I,JS,J2                                GR007660
C
C     WRITE(6,30)I,JS,J2                                GR007670
C
C     30 FORMAT(2X,' I =',I5,' J=',I5,' JS=',I2,' J2=',I2)  GR007680
CC***                                         GR007690

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END IF                               GR006600
RETURN                                GR006610
END                                     GR006620
C*****                                         GR006630
C                                         GR006640
C     INTEGER FUNCTION KDELTA(I,J)          GR006650
C                                         GR006660
C*****                                         GR006670
C                                         GR006680
C
IF (I.EQ.J) THEN                      GR006690
    KDELTA=1                           GR006700
ELSE
    KDELTA=0                           GR006720
END IF                                 GR006730
RETURN                                GR006740
END                                     GR006750
C*****                                         GR006760
C                                         GR006770
C     INTEGER FUNCTION IBINO(M,N)          GR006780
C                                         GR006790
C*****                                         GR006800
C                                         GR006810
C
IF (M.EQ.N)GO TO 11                  GR006820
IF ((M.LT.0).OR.(N.LT.0)) GO TO 10   GR006830
K=M-N                               GR006840
IF (K) 10,11,12                      GR006850
10  IBINO = 0                          GR006860
RETURN                                GR006870
11  IBINO = 1                          GR006880
RETURN                                GR006890
12  IF (N.EQ.0)GO TO 11               GR006900
IN = 1                                GR006910
IP = M                                GR006920
IQ = 1                                GR006930
DO 20 I=1,K                          GR006940
    IN = IN*IP/IQ                     GR006950
    IP = IP-1                         GR006960
    IQ = IQ +1                        GR006970
20  CONTINUE                           GR006980
    IBINO = IN                         GR006990
RETURN                                GR007000
END                                     GR007010
C*****                                         GR007020
C                                         GR007030
C     INTEGER FUNCTION KD1(N1,N2,JS,J2)    GR007040
C                                         GR007050
C*****                                         GR007060
C                                         GR007070
C
EXTERNAL KDELTA                      GR007080
KD1=KDELTA(0,N1)*KDELTA(2,J2)+KDELTA(0,N2)*KDELTA(1,J2)  GR007090
KD1=KD1*KDELTA(JS,J2)                 GR007100
RETURN                                GR007110
END                                     GR007120
C*****                                         GR007130
C                                         GR007140

```

```

        INTEGER FUNCTION KUPPER(N1,N2,JS,J2)           GR007150
C                                               GR007160
C*****                                         GR007170
C                                               GR007180
C
        EXTERNAL KDELTA                           GR007190
        K=N2+KDELTA(2,J2)*KDELTA(1,JS)-KDELTA(1,J2)*KDELTA(2,JS)   GR007200
        IF (N1.LE.K) THEN                         GR007210
          KUPPER=N1                                GR007220
        ELSE                                     GR007230
          KUPPER=K                                GR007240
        END IF                                    GR007250
        RETURN                                     GR007260
        END                                         GR007270
C*****                                         GR007280
C                                               GR007290
C
        INTEGER FUNCTION KLOWER(N1,N2,JS,J2)         GR007300
C                                               GR007310
C*****                                         GR007320
C                                               GR007330
C
        EXTERNAL KDELTA                           GR007340
        KLOWER=KDELTA(2,J2)+KDELTA(1,J2)*KDELTA(1,JS)    GR007350
        RETURN                                     GR007360
        END                                         GR007370
C*****                                         GR007380
C                                               GR007390
C
        SUBROUTINE COEFF(N1,N2,JS,J2,KD,KU,KL,KSUM)  GR007400
C                                               GR007410
C*****                                         GR007420
C                                               GR007430
C
        TO COMPUTE THE COEFFICIENTS FOR THE RSTAR TURM      GR007440
C                                               GR007450
C
        INPUTS:                                         GR007460
C
        N1 = NUMBER OF P TRIPS                         GR007470
C
        N2 = NUMBER OF S TRIPS                         GR007480
C
        JS = FINAL TRIP TYPE, 1 OR 2                  GR007490
C
        J2 = INITIAL TRIP TYPE, 1 OR 2                GR007500
C
        OUTPUT:                                         GR007510
C
        KD = FIRST COEFF                            GR007520
C
        KU = UPPER LIMIT IN SUMMATION               GR007530
C
        KL = LOWER LIMIT IN SUMMATION               GR007540
C
        KSUM = COEFF. IN SUMMATION, MUST BE DIMENSIONED IN CALLING  GR007550
C
        PROGRAM TO BE MAX(N1,N2)                      GR007560
C
C-----                                         GR007570
C                                               GR007580
C-----                                         GR007590
C                                               GR007600
C
        PROGRAM TESTCOEF                           GR007610
C
        DIMENSION KSUM(20)                          GR007620
CC***                                         GR007630
C
        1  WRITE(5,10)                             GR007640
C  10 FORMAT (2X,'TYPE K1,K2,JS,J2')          GR007650
C
        READ(5,*)I,J,JS,J2                        GR007660
C
        WRITE(6,30)I,J,JS,J2                      GR007670
C  30 FORMAT(2X,' I =',I5,' J=',I5,' JS=',I2,' J2=',I2)  GR007680
CC***                                         GR007690

```

```

IF (KASE.EQ.1) THEN                               GR008800
C      SOURCE AT TOP SURFACE                      GR008810
C
DO 200 I=1,3                                     GR008820
    YQS(I,1)=YRRP(1,1)*YUP(I,1)*YP1+YRRP(1,2)*YUP(I,2)*YP2   GR008830
    1      - YUM(I,1)*YP1                                GR008840
    YQS(I,2)=YRRP(2,1)*YUP(I,1)*YP1                                GR008850
    1      +YRRP(2,2)*YUP(I,2)*YP2 - YUM(I,2)*YP2          GR008860
    YQS(I,3)=(YRRP(3,3)*YUP(I,3) - YUM(I,3))*YP2           GR008870
200    CONTINUE                                    GR008880
C
C
ELSE IF (KASE.EQ.2) THEN                         GR008890
C      SOURCE AT BOTTOM SURFACE                     GR008900
C
DO 300 I=1,3                                     GR008910
    YQS(I,1)=YRRM(1,1)*YUM(I,1)*YP1+YRRM(1,2)*YUM(I,2)*YP2   GR008920
    1      - YUP(I,1)*YP1                                GR008930
    YQS(I,2)=YRRM(2,1)*YUM(I,1)*YP1                                GR008940
    1      +YRRM(2,2)*YUM(I,2)*YP2 - YUP(I,2)*YP2          GR008950
    YQS(I,3)=(YRRM(3,3)*YUM(I,3) - YUP(I,3))*YP2           GR008960
300    CONTINUE                                    GR008970
C
ENDIF                                         GR008980
RETURN                                         GR008990
END                                           GR009000
C ****
C SUBROUTINE EPIDIS(A,T,U)                      GR009010
C ****
C THIS PROGRAM CALCULATES THE VERTICAL DISPLACEMENT     GR009020
C AT THE EPICENTER OF A LARGE PLATE DUE TO AN UNIT       GR009030
C STEP FUNCTION VERTICAL FORCE.                         GR009040
C DOUBLE PRECISION IS USED.                           GR009050
C
C INPUT PARAMETERS :                                 GR009060
C
C A = SHEAR WAVE SPEED / LONGITUDINAL WAVE SPEED     GR009070
C   e.g. ALUMINUM : A=0.495                            GR009080
C T = NORMALIZED TIME = ACTUAL TIME *SHEAR WAVE SPEED / PLATE   GR009090
C               THICKNESS                           GR009100
C   e.g. SHEAR WAVE SPEED ~ 3 mm /micro second        GR009110
C
C OUTPUT PARAMETERS :                               GR009120
C
C U = NORMALIZED DISPLACEMENT = ACTUAL DISPLACEMENT * 3.14159   GR009130
C   * SHEAR MODULUS * PLATE THICKNESS / FORCE         GR009140
C
C SUBROUTINES REQUIRED:                            GR009150
C
C QPARA(T,A), AND FUNCTION BINO                  GR009160
C

```

|  |          |
|--|----------|
| C-----   | GR009350 |
| C  | GR009360 |
| C THE PROGRAM MAY BE SET UP AS THE FOLLOWINGS:             | GR009370 |
| C  | GR009380 |
| C- PROGRAM EPICT   | GR009390 |
| C- DOUBLE PRECISION A,DT,DELTAT,DU(1024)                   | GR009400 |
| C- A = 0.584D0   | GR009410 |
| C- NPT=1024  | GR009420 |
| C- DELTAT=0.00277D0  | GR009430 |
| C- DO 100 I=1,NPT  | GR009440 |
| C- DT=DELTAT*DBLE(REAL(I))                                 | GR009450 |
| C- CALL EPIDIS(A,DT,DU(I))                                 | GR009460 |
| C-100 CONTINUE   | GR009470 |
| C- WRITE(7) DU   | GR009480 |
| C- STOP  | GR009490 |
| C- END   | GR009500 |
| C  | GR009510 |
| C-----   | GR009520 |
| C  | GR009530 |
| IMPLICIT DOUBLE PRECISION (Q)                              | GR009540 |
| DOUBLE PRECISION U,T,A,TA,DXM                              | GR009550 |
| INTEGER BINO   | GR009560 |
| COMMON /BLOCK/ QN1,QN2,QQ,QP,QM,QPP,QPS,QSP,QSS,QPHI       | GR009570 |
| C  | GR009580 |
| C NRAY = MAX. POSSIBLE NUMBER OF P RAYS                    | GR009590 |
| C NRAY2 = MAX. POSSIBLE NUMBER OF S RAYS                   | GR009600 |
| NRAY = 20  | GR009610 |
| NRAY2 = 12   | GR009620 |
| U = 0.0  | GR009630 |
| DO 300 NA = 1,NRAY   | GR009640 |
| DO 200 NB = 1,NRAY2  | GR009650 |
| N1 = NA-1  | GR009660 |
| N2 = NB-1  | GR009670 |
| C     N1+N2 MUST BE AN ODD NUMBER                          | GR009680 |
| IF ((FLOAT(N1+N2)/2.).EQ.FLOAT((N1+N2)/2))GO TO 200        | GR009690 |
| TA = N2 + N1*A   | GR009700 |
| IF (T.LE.TA) GO TO 200                                     | GR009710 |
| QN1 = N1   | GR009720 |
| QN2 = N2   | GR009730 |
| C  | GR009740 |
| CALL QPARA(T,A)  | GR009750 |
| C  | GR009760 |
| NTERM = N2   | GR009770 |
| IF (N1.LT.N2) NTERM=N1                                     | GR009780 |
| NTERM=NTERM+1  | GR009790 |
| QSUM = 0.0   | GR009800 |
| DO 100 MT = 1,NTERM  | GR009810 |
| MM = MT-1  | GR009820 |
| QFPP = BINO(N1+1,MM+2)*BINO(N2+1,MM+1)                     | GR009830 |
| QFPS = BINO(N2+1,MM+1)*BINO(N1+1,MM+1)                     | GR009840 |
| QFSS = BINO(N2+1,MM+2)*BINO(N1+1,MM+1)                     | GR009850 |
| MO = N1 + N2 - 1 - 2*MM                                    | GR009860 |
| C  | GR009870 |
| QSUM = ((-1.)**N2)*  | GR009880 |
| 1 ((QFPP*QPP-QFSS*QSS)*(DXM(QO,MO)*DXM((QM*QP*(-1.)),MM))) | GR009890 |

```

2      +QFPS*(QPS*DXM(QO,(MO+1))*DXM((( -1.)*QP),MM)*DXM(QM,MM)
3      -QSP*DXM(QO,(MO+1))*DXM((( -1.)*QM),MM)*DXM(QP,MM)))
4      +QSUM

C
100    CONTINUE
        U = U + QSUM/QPHI
200    CONTINUE
300    CONTINUE
        RETURN
        END

C
C ****
C
C SUBROUTINE QPARA(T,A)
C
C ****
C
C IMPLICIT DOUBLE PRECISION (Q)
C DOUBLE PRECISION T,A,B,AC,BAC,XX,X,SX,SAX
C COMMON /BLOCK/ QN1,QN2,QO,QP,QM,QPP,QPS,QSP,QSS,QPHI
C
C CALCULATE X
C
        Q1=QN1*QN1
        Q2=QN2*QN2
        IF(QN1.EQ.0.00) THEN
            XX=(T*T-Q2)/Q2
        ELSE IF (QN2.EQ.0.00) THEN
            XX=(T*T-Q1*A*A)/Q1
        ELSE
            B = ((Q1+Q2)*T*T -(Q1*A*A-Q2)*(Q1-Q2))/((Q1-Q2)**2)
            AC = ((Q1*A*A-Q2)**2+T*T*(T*T-2.*Q1*A*A-2.*Q2))
            1   /((Q1-Q2)**2)
            BAC =B*B -AC
            IF (BAC.LE.0.0) BAC = 0.0
            XX = B-DSQRT(BAC)
            END IF
            X = DSQRT(XX)
            SX = DSQRT(1.+XX)
            SAX = DSQRT(A*A+XX)

C
C CALCULATE R'S, Q'S, AND QPHI
C
        QA= (1.+2.*XX)**2
        QB= 4.*SAX*SX*XX
        QO= (QA+QB)/(QA-QB)
        QP= -4.*X*A*SX*(1.+2.*XX)/(QA-QB)
        QM= 4.*X*SAX*(1.+2.*XX)/((QA-QB)*A)
        QPP= 2.*SAX*QA/((QA-QB)**2)
        QPS= SAX/(QA-QB)
        QSP=-SAX/(QA-QB)
        QSS= -2.*SAX*QB/((QA-QB)**2)
        QPHI= 2.*(QN1/SAX + QN2/SX)
        RETURN
        END

```

```

C                               GR010450
C ***** *****
C                               GR010460
C                               GR010470
C           INTEGER FUNCTION BINO(J1,J2)          GR010480
C ***** *****
C                               GR010490
C                               GR010500
C                               GR010510
C
C           M = J1-2                           GR010520
C           N = J2-2                           GR010530
C           IF ((M.EQ.N).OR.(N.EQ.0)) THEN      GR010540
C               BINO=1                         GR010550
C               RETURN                         GR010560
C           ELSE IF ((M.LT.0).OR.(N.LT.0).OR.(M.LT.N)) THEN   GR010570
C               BINO = 0                      GR010580
C               RETURN                         GR010590
C           ELSE                           GR010600
C               K=M-N                         GR010610
C               IN = 1                          GR010620
C               IP = M                          GR010630
C               IQ = 1                          GR010640
C               DO 20 I=1,K                   GR010650
C                   IN = IN*IP/IQ            GR010660
C                   IP = IP-1              GR010670
C                   IQ = IQ +1             GR010680
20         CONTINUE                         GR010690
C           BINO = IN                         GR010700
C           RETURN                           GR010710
C           ENDIF                           GR010720
C           END                             GR010730
C ***** *****
C                               GR010740
C                               GR010750
C           FUNCTION DXM(D,M)                GR010760
C                               GR010770
C ***** *****
C                               GR010780
C                               GR010790
C           THE FUNCTION DXM(D,M) IS        GR010800
C THE SAME AS D**M. BOTH D AND DXM MUST BE DECLARED IN    GR010810
C THE MAIN PROGRAM AS DOUBLE PRECISION VARIABLES.          GR010820
C                               GR010830
C           DOUBLE PRECISION DXM,D          GR010840
C           IF (M.EQ.0) DXM=1.0D0          GR010850
C           IF (M.GT.0) GO TO 11          GR010860
C           IF (M.LT.0) GO TO 22          GR010870
C           RETURN                         GR010880
C                               GR010890
11         DXM=1.0D0                         GR010900
C           DO 10 I=1,M                  GR010910
C           DXM=DXM*D                  GR010920
10         CONTINUE                         GR010930
C           RETURN                           GR010940
C                               GR010950
22         DXM=1.0D0                         GR010960
C           N=-M                          GR010970
C           DO 20 I=1,N                  GR010980
C           DXM=DXM/D                  GR010990

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|        |   |          |
|--------|---|----------|
| 20     | CONTINUE  | GR011000 |
|        | RETURN  | GR011010 |
| C      | END   | GR011020 |
| C***** |   | GR011030 |
| C      | SUBROUTINE SFWAVE(ALPHA,XR,TTIME,INDEX,DISPL)               | GR011040 |
| C***** |   | GR011050 |
| C      | SFWAVE  | GR011060 |
| C      | SUBROUTINE TO COMPUTE THE SURFACE DISPLACEMENT              | GR011070 |
| C      | INPUTS:   | GR011080 |
| C      | ALPHA=RATIO OF SHEAR WAVE SPEED TO LONG. WAVE SPEED         | GR011090 |
| C      | XR =DISTANCE BETWEEN SOURCE AND DETECTOR                    | GR011100 |
| C      | =ACTUAL DIST./H   | GR011110 |
| C      | TTIME =NONDIMENSIONALIZED TTIME                             | GR011120 |
| C      | =ACTUAL TIME*SHEAR WAVE SPEED / H                           | GR011130 |
| C      | INDEX=SUBSCRIPT OF THE GREEN'S FUNCTION                     | GR011140 |
| C      | e.g. 11,22,33,12,13 or 131,113, etc.                        | GR011150 |
| C      |   | GR011160 |
| C      | OUTPUT:   | GR011170 |
| C      | DISPL=NONDIMENSIONALIZED DISPLACEMENT                       | GR011180 |
| C      | =ACTUAL DISPL.* PI * SHEAR MODULUS * H / FORCE              | GR011190 |
| C      | NOTE:   | GR011200 |
| C      | THE PARAMETERS IN COMMON BLKS 0,1, & 5 SHOULD BE ENTERED    | GR011210 |
| C      | BEFORE THE FIRST CALL OF THIS SUBROUTINE,                   | GR011220 |
| C      | I. E. CALL INIT(ALPHA,XR,ZD,INDEX) FIRST.                   | GR011230 |
| C      |   | GR011240 |
| C      | SUBROUTINES REQUIRED:                                       | GR011250 |
| C      |   | GR011260 |
| C      | DSFINT,DQR,QU,COMMON,RAYRT & INTEGRATION ROUTINES           | GR011270 |
| C      |   | GR011280 |
| C      | REMARK: THE COMPUTATION IS DONE IN Y-PLANE, 4th QUARD. ON A | GR011290 |
| C      | THREE LEGGED U-PATH TO REPLACE THE INTEGRATION FROM         | GR011300 |
| C      | -V TO V.  | GR011310 |
| C      |   | GR011320 |
| C      |   | GR011330 |
| C      |   | GR011340 |
| C      |   | GR011350 |
| C      |   | GR011360 |
| C      |   | GR011370 |
| C      | IMPLICIT DOUBLE PRECISION (D)                               | GR011380 |
| C      | IMPLICIT DOUBLE COMPLEX (Y)                                 | GR011390 |
| C      | COMMON /BLK0/DXD,DT,DZ,DA                                   | GR011400 |
| C      | COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA                      | GR011410 |
| C      | COMMON /BLK5/YR,Y1,Y2                                       | GR011420 |
| C      |   | GR011430 |
| C      | N1=0  | GR011440 |
| C      | N2=0  | GR011450 |
| C      | DYR=DBLE(YR)  | GR011460 |
| C      | DT=DBLE(TTIME)  | GR011470 |
| C      | DZERO=0.0D0   | GR011480 |
| C      | DMONEH=-0.5D0   | GR011490 |
| C      | DSQRYR=DSQRT(DYR)   | GR011500 |
| C      | RWSPD=DSQRYR  | GR011510 |
| C      |   | GR011520 |
| C      | DPF=1.2D0/DBLE(ALPHA)                                       | GR011530 |
| C      | DV=DXD/DT   | GR011540 |

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        IF (TTIME.LE.(ALPHA*XR)) THEN                                GR011550
          DISPL=0.0D0                                                 GR011560
          RETURN                                                 GR011570
        ELSE
          DISPL=0.D0                                                 GR011580
          DISPL=DISPL+DSFING(DZERO,DMONEH,1)                         GR011590
          2           +DSFING(DV,DPF,2)                               GR011600
          3           +DSFING( DMONEH,DZERO,3)                         GR011610
        END IF                                                 GR011620
      END
C*****                                                       GR011630
C                                                       GR011640
C DOUBLE PRECISION FUNCTION DSFING(DINIT,DEND,IPATH)       GR011650
C                                                       GR011660
C*****                                                       GR011670
C                                                       GR011680
C*****                                                       GR011690
C                                                       GR011700
C IMPLICIT DOUBLE PRECISION (D)                             GR011710
C                                                       GR011720
C DOUBLE PRECISION EPS,R,E,W(50,6),FMIN,FMAX,F             GR011730
C INTEGER NINT,NMAX,KF,IFLAG                               GR011740
C LOGICAL RST                                              GR011750
C                                                       GR011760
C                                                       GR011770
C EXTERNAL DQR1,DQR2,DQR3                                 GR011780
C                                                       GR011790
C EPS = 1.D-8                                              GR011800
C RST = .FALSE.                                            GR011810
C NMAX = 50                                                GR011820
C   NINT = 1                                               GR011830
C   W(1,1) = DINIT                                         GR011840
C   W(2,1) = DEND                                           GR011850
C IF (IPATH.EQ.1) THEN                                     GR011860
C   CALL DGLQ1(DQR1,DINIT,DEND,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX, GR011870
C +   KF,IFLAG)                                            GR011880
C ELSE IF (IPATH.EQ.2)THEN                                 GR011890
C   CALL DGLQ1(DQR2,DINIT,DEND,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX, GR011900
C +   KF,IFLAG)                                            GR011910
C ELSE IF (IPATH.EQ.3) THEN                                 GR011920
C   CALL DGLQ1(DQR3,DINIT,DEND,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX, GR011930
C +   KF,IFLAG)                                            GR011940
C ELSE
C PAUSE 'DSFING: IPATH .NE. 1,2, OR 3'                   GR011950
C END IF                                                 GR011960
C                                                       GR011970
C                                                       GR011980
C D2PEI=8.D0*DASIN(1.D0)                                 GR011990
C DSFING=-R/D2PEI                                         GR012000
C RETURN                                                 GR012010
C END                                                 GR012020
C*****                                                       GR012030
C                                                       GR012040
C DOUBLE PRECISION FUNCTION DQR1(DL)                      GR012050
C                                                       GR012060
C*****                                                       GR012070
C IMPLICIT DOUBLE PRECISION (D)                           GR012080
C EXTERNAL DQR                                         GR012090

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DQR1=DQR(DL,1)                               GR012100
RETURN                                         GR012110
END                                            GR012120
C*****                                         GR012130
C                                         GR012140
C     DOUBLE PRECISION FUNCTION DQR2(DL)      GR012150
C                                         GR012160
C*****                                         GR012170
IMPLICIT DOUBLE PRECISION (D)               GR012180
EXTERNAL DQR                                GR012190
DQR2=DQR(DL,2)                               GR012200
RETURN                                         GR012210
END                                            GR012220
C*****                                         GR012230
C                                         GR012240
C     DOUBLE PRECISION FUNCTION DQR3(DL)      GR012250
C                                         GR012260
C*****                                         GR012270
IMPLICIT DOUBLE PRECISION (D)               GR012280
EXTERNAL DQR                                GR012290
DQR3=DQR(DL,3)                               GR012300
RETURN                                         GR012310
END                                            GR012320
C*****                                         GR012330
C                                         GR012340
C     DOUBLE PRECISION FUNCTION DQR(DL,IPATH)  GR012350
C                                         GR012360
C*****                                         GR012370
C                                         GR012380
IMPLICIT DOUBLE COMPLEX (Y)                 GR012390
IMPLICIT DOUBLE PRECISION (D)               GR012400
DIMENSION YQD(3,3),YUP(3,3),YQK(3),YFEE(3)  GR012410
COMMON /BLKO/DXD,DT,DZ,DA                  GR012420
COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA    GR012430
COMMON /BLK2/YD,Y,YP,YQ,YRO,YRP,YRM,YDELTA,  GR012440
1   YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2  GR012450
2   ,YDPHI                                         GR012460
COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO  GR012470
C                                         GR012480
C                                         GR012490
DV=DXD/DT                                    GR012500
IF (IPATH.EQ.1) THEN                         GR012510
  Y= DCMPLX( DV,DL)                         GR012520
ELSE IF (IPATH.EQ.2) THEN                     GR012530
  Y= DCMPLX(DL,-0.5D0)                      GR012540
ELSE IF (IPATH.EQ.3) THEN                     GR012550
  DPF=1.2D0/DA                             GR012560
  Y= DCMPLX(DPF,DL)                         GR012570
ELSE                                         GR012580
  PAUSE 'DRQ: IPATH ?'                      GR012590
ENDIF                                         GR012600
CALL SFCOM(DL)                               GR012610
CALL QD(YQD, IDUMMY)                         GR012620
CALL QU(YUP, IDUMMY)                         GR012630
C                                         GR012640

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YFEE(1)=YAA/(YP*YD) GR012650
YFEE(2)=YONE/(YQ*YD) GR012660
YFEE(3)=YONE/(YQ*YD) GR012670
C GR012680
YQR = YZERO GR012690
C GR012700
IF (K.EQ.0) THEN GR012710
YQK(1)=YONE GR012720
YQK(2)=YONE GR012730
YQK(3)=YONE GR012740
ELSE IF (K.EQ.1) THEN GR012750
YQK(1)=YETA1/Y GR012760
YQK(2)=YETA1/Y GR012770
YQK(3)=YETA1/Y GR012780
ELSE IF (K.EQ.2) THEN GR012790
YQK(1)=YETA2/Y GR012800
YQK(2)=YETA2/Y GR012810
YQK(3)=YETA2/Y GR012820
ELSE IF (K.EQ.3) THEN GR012830
YQK(1)=-YP/Y GR012840
YQK(2)=-YQ/Y GR012850
YQK(3)=-YQ/Y GR012860
ELSE GR012870
PAUSE ' FUUNCTION DQR: WRONG INDEX, K .NE. 0,1,2, OR 3' GR012880
END IF GR012890
DO 100 I=1,3 GR012900
    YQR=YQR+YQD(M,I)*YUP(L,I)*YQK(I)*YFEE(I) GR012910
100    CONTINUE GR012920
C GR012930
IF ((IPATH.EQ.1).OR.(IPATH.EQ.3)) THEN GR012940
    YQR=YQR*YI/CDSQRT(DCMPLX(DV*DV,0.0D-00)-YY) GR012950
ELSE GR012960
    YQR=YQR/CDSQRT(DCMPLX(DV*DV,0.0D-00)-YY) GR012970
END IF GR012980
DQR=DBLE(YQR) GR012990
C GR013000
RETURN GR013010
END GR013020
C***** GR013030
C***** GR013040
C SUBROUTINE SFCOM(DL) GR013050
C GR013060
C***** GR013070
C GR013080
C TO COMPUTE PARAMETERS DEPENDING ON Y AND TO PUT IN GR013090
C A COMMON BLK2 GR013100
C GR013110
IMPLICIT DOUBLE COMPLEX (Y) GR013120
IMPLICIT DOUBLE PRECISION (D) GR013130
DOUBLE PRECISION ETA1,ETA2SQ,DD,AA GR013140
COMMON /BLKO/DXD,DT,DZ,DA GR013150
COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA GR013160
COMMON /BLK2/YD,YP,YQ,YR0,YRP,YRM,YDELTA,YPSQP1,YQSQP1,GR013170
1     YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2 GR013180
2     ,YDPHI GR013190

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COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO GR013200
C
AA=DA*DA GR013220
DV=DXD/DT GR013230
DN1=DBLE(REAL(N1)) GR013240
DN2=DBLE(REAL(N2)) GR013250
YZERO=DCMPLX(0.0D0,0.0D0) GR013260
YI=DCMPLX(0.0D0,1.0D0) GR013270
YAA=DCMPLX(AA,0.0D0) GR013280
YA=DCMPLX(DA,0.0D0) GR013290
YONE=DCMPLX(1.0D0,0.0D0) GR013300
YTWO=DCMPLX(2.0D0,0.0D0) GR013310
YFOUR=DCMPLX(4.0D0,0.0D0) GR013320
C
YD=-Y*DCMPLX(DT,0.0D0) GR013330
YETA1 = -Y/DCMPLX(DV,0.0D0) GR013340
YETA2=CDSQRT(YONE-YETA1*YETA1) GR013350
IF (DBLE(Y).LT.0.0D0) YETA2=-YETA2 GR013360
YY=Y*Y GR013370
YP=CDSQRT(YAA*YY-YONE) GR013380
YQ=CDSQRT(YY-YONE) GR013390
YPSQP1=YAA*YY GR013400
YQSQP1=YY GR013410
YQSQM1=YQ*YQ-YONE GR013420
YPSQM1=YP*YP-YONE GR013430
C
YSQRP1=Y*YA GR013440
YSQRQ1=Y GR013450
YDELTA=YQSQM1*YQSQM1+YFOUR*YP*YQ GR013460
YM=(YQSQM1*YQSQM1-YFOUR*YP*YQ) GR013470
YR0=YM/YDELTA GR013480
YRP=-(YFOUR*YP*YQSQM1)/(YA*YDELTA) GR013490
YRM=(YR0*YR0-YONE)/YRP GR013500
C
RETURN GR013510
END GR013520
C*****
SUBROUTINE QU(YUP, IDUMMY) GR013530
C*****
C TO COMPUTE QU(I,J) BASED ON EQ. 6.4 GR013540
C
IMPLICIT DOUBLE COMPLEX (Y) GR013550
COMMON /BLK0/DXD,DT,DZ,DA GR013560
COMMON /BLK1/KASE,M,L,K,N1,N2,D,DTHETA GR013570
COMMON /BLK2/YD,Y,YP,YQ,YR0,YRP,YRM,YDELTA,YPSQP1,YQSQP1,  GR013580
1 YPSQM1,YQSQM1,YSQRP1,YSQRQ1,YY,YA,YAA,YETA1,YETA2 GR013590
2 ,YDPHI GR013600
COMMON /BLK3/YI,YONE,YTWO,YTHREE,YFOUR,YEIGHT,YZERO GR013610
C
DIMENSION YUP(3,3) GR013620
C
IDUMMY=0 GR013630

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YUP(1,1)=YETA1/YSQRP1 GR013750
YUP(1,2)=-YETA1*YQ/YSQRQ1 GR013760
YUP(1,3)=-YETA2 GR013770
YUP(2,1)=YETA2/YSQRP1 GR013780
YUP(2,2)=-YETA2*YQ/YSQRQ1 GR013790
YUP(2,3)=YETA1 GR013800
YUP(3,1)=-YP/YSQRP1 GR013810
YUP(3,2)=-YONE/YSQRQ1 GR013820
YUP(3,3)=YZERO GR013830
C GR013840
RETURN GR013850
END GR013860
C***** GR013870
C GR013880
SUBROUTINE RAYRT(YR,YI1,YI2,ALPHA) GR013890
C GR013900
C***** GR013910
C GR013920
C TO SOLVE RAYLEIGH EQUITION GR013930
C GR013940
C INPUT: ALPHA = SHEAR WAVE SPEED / LONG. WAVE SPEED GR013950
C GR013960
C OUTPUT: YR,YI1,YI2 ARE THE ROOTS TO GR013970
C GR013980
C Y**Y*Y-8*Y**Y+8*(3-2*AA)*Y-16*(1-AA) = 0 GR013990
C GR014000
C DOUBLE COMPLEX YR,YI1,YI2 GR014010
DOUBLE PRECISION A,AA,RT1,RT2,RT3 GR014020
DIMENSION A(4),RT1(2),RT2(2),RT3(2) GR014030
C GR014040
IF((ALPHA.LE.0.) .OR. (ALPHA.GE.1.0)) THEN GR014050
  WRITE(6,*)'RAYRT: WRONG VALUE OF ALPHA' GR014060
  WRITE(6,*) ALPHA GR014070
  WRITE(6,*)'RAYRT: COMPUTATION PAUSED' GR014080
  PAUSE GR014090
ENDIF GR014100
C GR014110
AA=ALPHA*ALPHA GR014120
TOL= 1.E-37 GR014130
A(1) = 1.0 GR014140
A(2) = -8.0D0 GR014150
A(3) = 8.*(3.-2.*AA) GR014160
A(4) = -16.*(1.-AA) GR014170
C GR014180
CALL CUBIC(A,RT1,RT2,RT3,TOL,NOROOT) GR014190
C GR014200
C IF (ALPHA.LT.0.567) THEN GR014210
C GR014220
C ONLY ONE REAL ROOT GR014230
C GR014240
C ELSE GR014250
C GR014260
C ALL THREE ROOTS ARE REAL GR014270
C GR014280
IF ((RT1(1).LT.RT2(1)).AND.(RT1(1).LT.RT3(1)))THEN GR014290

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YR=DCMPLX(RT1(1),RT1(2)) GR014300
YI1=DCMPLX(RT2(1),RT2(2)) GR014310
YI2=DCMPLX(RT3(1),RT3(2)) GR014320
ELSEIF ((RT2(1).LT.RT1(1)).AND.(RT2(1).LT.RT3(1))) THEN GR014330
    YI1=DCMPLX(RT1(1),RT1(2)) GR014340
    YR =DCMPLX(RT2(1),RT2(2)) GR014350
    YI2=DCMPLX(RT3(1),RT3(2)) GR014360
ELSEIF ((RT3(1).LT.RT1(1)).AND.(RT3(1).LT.RT2(1)))THEN GR014370
    YI2=DCMPLX(RT1(1),RT1(2)) GR014380
    YI1=DCMPLX(RT2(1),RT2(2)) GR014390
    YR =DCMPLX(RT3(1),RT3(2)) GR014400
ENDIF GR014410
RETURN GR014420
END GR014430
C*****
C
SUBROUTINE TIMEARRI(A,R,ZD,NRAY,RCN,TA,CN) GR014440
C*****
C
C 9/27/82 GR014450
C
C
DIMENSION Z1(4),Z2(4), TA(NRAY),CN(NRAY,3),RCN(NRAY,3) GR014460
DOUBLE COMPLEX YR,YI1,YI2 GR014470
ITER=100 GR014480
NRAY=NRAY GR014490
I=0 GR014500
LM=1 GR014510
C Z = ZD+0.5 ; ZD refers to origin at the center of the plate GR014520
Z = ZD+0.5 GR014530
Z1(1)=0.0 GR014540
Z2(1)=Z GR014550
Z1(2)=Z GR014560
Z2(2)=0.0 GR014570
Z1(3)=-Z GR014580
Z2(3)=0.0 GR014590
Z1(4)=0.0 GR014600
Z2(4)=-Z GR014610
C
IF (Z.EQ.1.0) Z2(1)=0.0 GR014620
OREVEN=FLOAT(INT(Z))*0.5 GR014630
DO 100 J=1,21 GR014640
DO 200 K=1,12 GR014650
JJ=J-1 GR014660
KK=K-1 GR014670
IF ( (K.EQ.1).AND.(J.EQ.1) ) THEN GR014680
    IF(Z.EQ.1.0) THEN GR014690
C THREE SURFACE WAVE ARRIVALS: GR014700
        I=I+1 GR014710
        CN(I,1)=-1. GR014720
        CN(I,2)=0. GR014730
        CN(I,3)=R*A GR014740
        TA(I)=CN(I,3) GR014750
        I=I+1 GR014760
    ENDIF GR014770
ENDIF GR014780
ENDC GR014790

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CN(I,1)=-1.                               GR014850
CN(I,2)=-1.                               GR014860
CN(I,3)=R                                 GR014870
TA(I)=CN(I,3)                            GR014880
CALL RAYRT(YR,YI1,YI2,A)                 GR014890
I=I+1                                    GR014900
CN(I,1)=0.                                GR014910
CN(I,2)=0.                                GR014920
CN(I,3)=R/DSQRT(DBLE(YR))               GR014930
TA(I)=CN(I,3)                            GR014940
ENDIF                                     GR014950
GOTO 200                                 GR014960
ENDIF                                     GR014970
IF ( (Z.EQ.0.0).OR.(Z.EQ.1.0) ) LM=4    GR014980
IF ( ((FLOAT(JJ+KK))/2.0-OREVEN).EQ.FLOAT((JJ+KK)/2) )
1      GO TO 200                           GR015000
DO 300 LL=1,4,LM                         GR015010
  C1=JJ+Z1(LL)                           GR015020
  C2=KK+Z2(LL)                           GR015030
  IF ( (C1.LT.0.0) .OR. (C2.LT.0.0) ) GO TO 300   GR015040
  I=I+1                                  GR015050
  IF (I.GT.NRAY) GO TO 99                GR015060
  CALL TARRV (A,C1,C2,R,ITER,ERROR,T)     GR015070
  IF (ERROR.GT.1.E-6) WRITE (6,603) I,C1,C2,ERROR   GR015080
  CN(I,1)=C1                             GR015090
  CN(I,2)=C2                             GR015100
  CN(I,3)=T                             GR015110
  TA(I)=T                               GR015120
  IF (C1.GT.0.)GO TO 300                GR015130
  APJ=A*C2/SQRT(1.-A*A)                 GR015140
  IF(R.LE.APJ) GO TO 300                GR015150
  C2=-C2                                GR015160
  CALL TARRV (A,C1,C2,R,ITER,ERROR,T)     GR015170
  I=I+1                                  GR015180
  IF (I.GT.NRAY) GO TO 99                GR015190
  CN(I,1)=C1                             GR015200
  CN(I,2)=C2                             GR015210
  CN(I,3)=T                             GR015220
  TA(I)=T                               GR015230
300      CONTINUE                          GR015240
200      CONTINUE                          GR015250
100      CONTINUE                          GR015260
C
99      CONTINUE                          GR015270
      CALL RSORT(CN,TA,RCN,NRAY,3,0)        GR015280
C
603      FORMAT(2X,'ERROR IS LARGER THAN 1.E-6 ',I3,3F10.5)  GR015290
      RETURN                               GR015300
      END                                 GR015310
*****                                         GR015320
C                                         GR015330
C                                         GR015340
SUBROUTINE TARRV(A,C1,C2,R,ITER,ERR,TARR)  GR015350
C                                         GR015360
*****                                         GR015370
C                                         GR015380
*****                                         GR015390

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IMPLICIT DOUBLE PRECISION (D) GR015400
C
NUM = 0 GR015410
DT1=0.1570796326794901 GR015420
DT2=0.0D0 GR015430
DA=DBLE(A) GR015440
DAA=DA*DA GR015450
DN1=DBLE(C1) GR015460
DN2=DBLE(C2) GR015470
DR=DBLE(R) GR015480
IF (C1.EQ.0.0) GO TO 2 GR015490
IF (C2.EQ.0.0) GO TO 3 GR015500
GR015510
C
1 CONTINUE GR015520
NUM = NUM + 1 GR015530
DT=(DT1+DT2)/2.0D0 GR015540
DAASS=DAA*DSIN(DT)*DSIN(DT) GR015550
DAASS=1.0D0-DAASS GR015560
DCS2=DSQRT(DAASS) GR015570
DRT = DN1*DTAN(DT) + DN2*DA*DSIN(DT)/DCS2 GR015580
GR015590
C WRITE (6,20) NUM,DT,DRT,C1,C2 GR015600
IF (NUM.GE.ITER) GO TO 10 GR015610
IF (DABS(DRT-DR).LE.5.D-11) GO TO 10 GR015620
IF (DRT.LE.DR) DT2=DT GR015630
IF (DRT.GT.DR) DT1=DT GR015640
GO TO 1 GR015650
C
GR015660
C 20 FORMAT ('NUM= ',I3,' T= ',D17.10,' RT= ',D17.10,2(2X,F4.0)) GR015670
10 ITER=NUM GR015680
DTARR=DA*DN1/DCOS(DT) + DN2/DCS2 GR015690
DERR=DABS(DR-DRT) GR015700
ERR=DERR GR015710
TARR=DTARR GR015720
RETURN GR015730
C
GR015740
2 IF (C2.LT.0.0) GO TO 4 GR015750
DTARR=DSQRT(DR*DR + DN2*DN2) GR015760
TARR=DTARR GR015770
ERR=0.0 GR015780
RETURN GR015790
C
GR015800
3 DTARR=DSQRT(DR*DR+DN1*DN1)*DA GR015810
TARR=DTARR GR015820
ERR=0.0 GR015830
RETURN GR015840
C
GR015850
4 DTARR=DR*DA - DN2*DSQRT(1.D0-DAA) GR015860
C NOTE HERE DN2 < 0 GR015870
TARR=DTARR GR015880
ERR=0.0 GR015890
RETURN GR015900
C
GR015910
END GR015920
C***** GR015930
C GR015940

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SUBROUTINE CUBIC(A,RT1,RT2,RT3,TOL,NOROOT)                               GR015950
C*****                                                                     GR015960
C*****                                                                     GR015970
C*****                                                                     GR015980
C THIS SUBROUTINE FINDS THE ROOTS OF A CUBIC EQUITION                      GR015990
C
C INPUTS:                                                               GR016000
C     A      COEFFICIENTS, A(4)                                         GR016010
C
C OUTPUTS:                                                               GR016020
C     RT1,RT2,RT3   THREE ROOTS                                         GR016030
C             RT1(1)=REAL PART OF RT1, ETC.                                GR016040
C
C THE ROUTINE CALLS LINCGN,AND QUADRT                                       GR016050
C
C DOUBLE PRECISION A,B,C,RT1,RT2,RT3,ZERO,X,Y,ZZ,SQT3,RC27,ANG           GR016060
C DIMENSION A(4),RT1(2),RT2(2),RT3(2),B(2),C(4)                           GR016070
C
C ZERO=TOL/10.                                                               GR016080
C SQT3=DSQRT(3.0D0)                                                       GR016090
C RC27=1.0/27.0                                                             GR016100
C
C IF (DABS(A(1))-ZERO) 7,7,12                                              GR016110
7    CALL QUADRT (A(2),RT1,RT2,TOL,NOROOT)                                 GR016120
      RETURN
C
12 NOROOT=3                                                               GR016130
  IT=0
C
  DO 1 I=2,4                                                               GR016140
    A(I)=A(I)/A(1)                                                       GR016150
1 CONTINUE
  A(1)=1.0
C
  IF (DABS(A(2)).LE.ZERO) GOTO 2                                         GR016160
  NDA=3
  B(1)=1.0
  B(2)=-(A(2)/3.0)
  CALL LINCGN(A,NDA,B,C)                                                 GR016170
  IT=1
  GO TO 4
2 DO 3 I=1,4                                                               GR016180
    C(I)=A(I)
3 CONTINUE
C
4 X = C(4)*C(4)*0.25+(C(3)**3)*RC27                                     GR016190
  IF (X.LT.0.0) GO TO 9
  X = DSQRT(X)
  Y = -(C(4)*0.5)
  I=1
  RT1(I) =Y+X
C
5 N = 0
  IF(RT1(I).LT.0.0) N=1
  IF (DABS(RT1(I)).LE.ZERO) GO TO 6

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      RT1(I) = DABS(RT1(I))**(1./3.)
      IF (N.EQ.1) RT1(I) = -RT1(I)
6 IF (I.EQ.2) GOTO 8
      I = 2
      RT1(I) = Y-X
      GO TO 5
C
8 RT2(2) = ((RT1(1) - RT1(2))*0.5)*SQRT3
      RT1(1) = RT1(1) + RT1(2)
      RT1(2) = 0.0
      RT2(1) = -RT1(1)*0.5
      RT3(1) = RT2(1)
      RT3(2) = -RT2(2)
      GO TO 11
C
9 ZZ = DABS(C(3))
      X = -(C(4)*0.5)/DSQRT((ZZ**3)*RC27 )
      ANG = DACOS(X)
      Y = 2.0*(DSQRT(ZZ/3.0))
      ANG=ANG/3.0
      RT1(1) = Y*DCOS(ANG)
      RT2(1) = Y*DCOS(ANG + 2.0943951024D0)
      RT3(1) = Y*DCOS(ANG + 4.1887902047D0)
      RT1(2) = 0.0
      RT2(2) = 0.0
      RT3(2) = 0.0
C
11 IF (IT.EQ.1) THEN
      RT1(1) = RT1(1) + B(2)
      RT2(1) = RT2(1) + B(2)
      RT3(1) = RT3(1) + B(2)
      ELSE
      ENDIF
      RETURN
      END
*****
C
      SUBROUTINE LINCNG(A,NDA,B,C)
C
*****
C
C      TO MAKE A LINEAR CHANGE OF VARIABLES
C      IN A GIVEN POLYNOMIAL
C
      DOUBLE PRECISION A,B,C,BIPWR
      DIMENSION A(1),B(2),C(1)
C
      BIPWR = 1.0D0
      NZ = NDA +1
C
      DO 4 I=1,NZ
          C(I) = A(I)
4 CONTINUE
C
      6 DO 8 I=2,NZ

```

GR016500  
GR016510  
GR016520  
GR016530  
GR016540  
GR016550  
GR016560  
GR016570  
GR016580  
GR016590  
GR016600  
GR016610  
GR016620  
GR016630  
GR016640  
GR016650  
GR016660  
GR016670  
GR016680  
GR016690  
GR016700  
GR016710  
GR016720  
GR016730  
GR016740  
GR016750  
GR016760  
GR016770  
GR016780  
GR016790  
GR016800  
GR016810  
GR016820  
GR016830  
GR016840  
GR016850  
GR016860  
GR016870  
GR016880  
GR016890  
GR016900  
GR016910  
GR016920  
GR016930  
GR016940  
GR016950  
GR016960  
GR016970  
GR016980  
GR016990  
GR017000  
GR017010  
GR017020  
GR017030  
GR017040

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C(I) = C(I) + C(I-1)*B(2) GR017050
8 CONTINUE GR017060
C GR017070
C C(NZ) = C(NZ)*BIPWR GR017080
C NZ = NZ - 1 GR017090
C BIPWR = BIPWR*B(1) GR017100
C IF (NZ.GT.1) GO TO 6 GR017110
C C(1) = C(1)*BIPWR GR017120
C RETURN GR017130
C END GR017140
C***** GR017150
C SUBROUTINE DGLQ1(F,A,B,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX,KF,IFLAG)GR017170
C GR017180
C***** GR017190
C GR017200
C GR017210
C *** ALL REAL VARIABLES ARE TYPED DOUBLE PRECISION GR017220
C *** 9/20/82 NH GR017230
C GR017240
C GR017250
C DGLQ1 IS A SUBROUTINE FOR THE AUTOMATIC EVALUATION GR017260
C OF DEFINITE INTEGRALS OF A USER DEFINED FUNCTION GR017270
C OF ONE VARIABLE PROVIDING FLEXIBLE USAGE. GR017280
C GR017290
C FOR AN EASY TO USE VERSION SEE SUBROUTINE DGLQ. GR017300
C GR017310
C CAPABILITIES OF DGLQ1 (IN ADDITION TO THOSE OF DGLQ) GR017320
C INCLUDE: GR017330
C ABILITY TO RESTART A CALCULATION TO GREATER GR017340
C ACCURACY WITHOUT PENALTY... GR017350
C ABILITY TO SPECIFY AN INITIAL PARTITION OF GR017360
C THE INTEGRATION INTERVAL... GR017370
C ABILITY TO INCREASE THE WORK SPACE TO HANDLE GR017380
C MORE DIFFICULT PROBLEMS... GR017390
C OUTPUT OF LARGEST/SMALLEST INTEGRAND VALUE FOR GR017400
C APPLICATIONS SUCH AS GRAPHING... GR017410
C GR017420
C ARGUMENTS IN THE CALL SEQUENCE GR017430
C GR017440
C F (INPUT) THE NAME OF YOUR INTEGRAND FUNCTION. GR017450
C THIS NAME MUST APPEAR IN AN EXTERNAL STATEMENT GR017460
C IN ANY PROGRAM WHICH CALLS DGLQ1. GR017470
C YOU MUST WRITE F IN THE FORM GR017480
C FUNCTION F(X) GR017490
C F=(EVALUATE INTEGRAND AT THE POINT X) GR017500
C RETURN GR017510
C END GR017520
C A GR017530
C B (INPUT) ENDPOINTS OF INTEGRATION INTERVAL GR017540
C EPS (INPUT) ACCURACY TO WHICH THE INTEGRAL IS TO BE CALCULAGR017550
C DGLQ1 WILL TRY TO ACHIEVE RELATIVE ACCURACY, GR017560
C E.G. SET EPS=.01 FOR 2 DIGITS, .001 FOR 3, ETGR017570
C R (OUTPUT) THE ESTIMATE OF THE INTEGRAL GR017580
C E (OUTPUT) THE ESTIMATE OF THE ABSOLUTE ERROR IN R. GR017590

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C NINT (INPUT) GR017600  
 C OUTPUT) GR017610  
 C AS AN OUTPUT QUANTITY, NINT GIVES THE GR017620  
 C NUMBER OF SUBINTERVALS IN THE FINAL GR017630  
 C PARTITION OF [A,B]. GR017640  
 C AS AN INPUT QUANTITY, NINT MUST BE SET TO GR017650  
 C THE NUMBER OF SUBINTERVALS IN THE INITIAL GR017660  
 C PARTITION OF [A,B]. FOR MOST PROBLEMS GR017670  
 C THIS IS JUST 1, THE INTERVAL [A,B] ITSELF. GR017680  
 C NINT IS USEFUL IF YOU WOULD LIKE TO HELP GR017690  
 C DGLQ1 LOCATE A DIFFICULT SPOT ON [A,B]. GR017700  
 C IN THIS REGARD NINT IS USED ALONG GR017710  
 C WITH THE ARRAY W (SEE BELOW). IF YOU SET GR017720  
 C NINT=1 IT IS NOT NECESSARY TO BE CONCERNED GR017730  
 C WITH W, EXCEPT THAT IT MUST BE DIMENSIONED...GR017740  
 C AS AN EXAMPLE OF MORE GENERAL APPLICATIONS, GR017750  
 C IF [A,B]=[0,1] BUT THE INTEGRAND JUMPS AT 0.3GR017760  
 C IT WOULD BE WISE TO SET NINT=2 AND THEN SET GR017770  
 C W(1,1)=0.0 (LEFT ENDPOINT) GR017780  
 C W(2,1)=0.3 (SINGULAR POINT) GR017790  
 C W(3,1)=1.0 (RIGHT ENDPOINT) GR017800  
 C IF YOU SET NINT GREATER THAN 1, BE SURE TO GR017810  
 C CHECK THAT YOU HAVE ALSO SET GR017820  
 C W(1,1)=A AND W(NINT+1,1)=B GR017830  
 C RST (INPUT) A LOGICAL VARIABLE (E.G. TRUE OR FALSE) GR017840  
 C SET RST=.FALSE. FOR INITIAL CALL TO DGLQ1 GR017850  
 C SET RST=.TRUE. FOR A SUBSEQUENT CALL, GR017860  
 C E.G. ONE FOR WHICH MORE ACCURACY IS GR017870  
 C DESIRED (SMALLER EPS). A RESTART ONLY GR017880  
 C MAKES SENSE IF THE PRECEDING CALL RETURNED GR017890  
 C WITH A VALUE OF IFLAG (SEE BELOW) LESS THAN GR017900  
 C ON A RESTART YOU MAY NOT CHANGE THE VALUES OGR017910  
 C ANY ARGUMENTS IN THE CALL SEQUENCE, EXCEPT EGRO17920  
 C W(NMAX,6) W IS AN ARRAY USED FOR SCRATCH STORAGE BY DGLQ1.GR017930  
 C YOU M U S T INCLUDE A DIMENSION STATEMENT IN GR017940  
 C YOUR CALLING PROGRAM TO ALLOCATE THIS STORAGE. GR017950  
 C THIS SHOULD BE OF THE FORM GR017960  
 C DIMENSION W(NMAX,6) GR017970  
 C WHERE NMAX IS AN INTEGER. AN ADEQUATE VALUE OF GR017980  
 C NMAX IS 50. GR017990  
 C NMAX (INPUT) AN INTEGER EQUAL TO THE FIRST SUBSCRIPT IN THE GR018000  
 C DIMENSION STATEMENT FOR THE ARRAY W. THIS IS GR018010  
 C ALSO EQUAL TO THE MAXIMUM NUMBER OF SUBINTERVALS GR018020  
 C PERMITTED IN THE INTERNAL PARTITION OF [A,B]. GR018030  
 C A VALUE OF 50 IS AMPLE FOR MOST PROBLEMS. GR018040  
 C FMIN GR018050  
 C FMAX (OUTPUT) THE SMALLEST AND LARGEST VALUES OF THE INTEGRAL GR018060  
 C WHICH OCCURRED DURING THE CALCULATION. THE GR018070  
 C ACTUAL INTEGRAND RANGE ON [A,B] MAY, OF COURSGR018080  
 C BE GREATER BUT PROBABLY NOT BY MORE THAN 10%.GR018090  
 C KF (OUTPUT) THE ACTUAL NUMBER OF INTEGRAND EVALUATIONS USEDGR018100  
 C BY DGLQ1 TO APPROXIMATE THIS INTEGRAL. KF GR018110  
 C WILL ALWAYS BE AT LEAST 30. GR018120  
 C IFLAG (OUTPUT) TERMINATION FLAG...POSSIBLE VALUES ARE GR018130  
 C 0 NORMAL COMPLETION, E SATISFIES GR018140

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C           E<EPS AND E<EPS*DABS(R)                      GR018150
C           1   NORMAL COMPLETION, E SATISFIES               GR018160
C                   E<EPS, BUT NOT RELATIVE ERROR REQUEST    GR018170
C           2   NORMAL COMPLETION, E SATISFIES               GR018180
C                   E<EPS*DABS(R)                         GR018190
C           3   NORMAL COMPLETION BUT EPS WAS TOO SMALL TO   GR018200
C                   SATISFY ABSOLUTE OR RELATIVE ERROR REQUEST. GR018210
C           4   ABORTED CALCULATION BECAUSE OF SERIOUS ROUNDING GR018220
C                   ERROR. PROBABLY E AND R ARE CONSISTENT.     GR018230
C           5   ABORTED CALCULATION BECAUSE OF INSUFFICIENT STORAGR018240
C                   R AND E ARE CONSISTENT. PERHAPS INCREASING NMAGR018250
C                   WILL PRODUCE BETTER RESULTS.                 GR018260
C           6   ABORTED CALCULATION BECAUSE OF SERIOUS DIFFICULTIGR018270
C                   MEETING YOUR ERROR REQUEST.                  GR018280
C           7   ABORTED CALCULATION BECAUSE EITHER EPS, NINT OR NGR018290
C                   HAS BEEN SET TO AN ILLEGAL VALUE.          GR018300
C           8   ABORTED CALCULATION BECAUSE YOU SET NINT>1 BUT FOGR018310
C                   TO SET W(1,1)=A AND W(NINT+1,1)=B            GR018320
C                                         GR018330
C T Y P I C A L   P R O B L E M   S E T   U P             GR018340
C                                         GR018350
C DIMENSION W(50,6)                                     GR018360
C LOGICAL RST                                         GR018370
C EXTERNAL F                                           GR018380
C A=0.0                                              GR018390
C B=1.0                                              GR018400
C W(1,1)=A                                         GR018410
C W(2,1)=.3      [SET INTERNAL PARTITION POINT AT .3]  GR018420
C W(3,1)=B                                         GR018430
C NINT=2      [INITIAL PARTITION HAS 2 INTERVALS]       GR018440
C RST=.FALSE.                                         GR018450
C EPS=.001                                            GR018460
C NMAX=50                                            GR018470
C                                         GR018480
C CALL DGLQ1(F,A,B,EPS,R,E,NINT,RST,W,NMAX,FMIN,FMAX,KF,IFLAG) GR018490
C                                         GR018500
C           1       A,B,EPS,R,E,NINT,FMIN,FMAX,KF,IFLAG        GR018510
C           IF(EPS.EQ. .0001 .OR. IFLAG.GE.3)STOP              GR018520
C           RST=.TRUE                                         GR018530
C           EPS=.0001      [ASK FOR ANOTHER DIGIT]          GR018540
C           GO TO 1                                         GR018550
C           END                                             GR018560
C           FUNCTION F(X)                                GR018570
C           IF(X.LT. .3)                                 GR018580
C           1 THEN                                         GR018590
C               F=X**(.2D0)* ALOG(X)                     GR018600
C           ELSE                                         GR018610
C               F=DSIN(X)                                GR018620
C           ENDIF                                         GR018630
C           RETURN                                         GR018640
C           END                                             GR018650
C                                         GR018660
C E N D   O F   D O C U M E N T A T I O N             GR018670
C                                         GR018680
C INTEGER C                                         GR018690

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DOUBLE PRECISION A,B,E,EB,EPMACH,EPS,FMAX,FMAXL,FMAXR,FMIN,FMINL   GR018700
1           ,FMINR,FMN,FMX,R,RAB,RABS,RAV,SIGN,T,TE                 GR018710
2           ,TE1,TE2,TR,TR1,TR2,UFLOW,W,XM,F                         GR018720
DIMENSION W(NMAX,6)                                                 GR018730
EXTERNAL F                                                       GR018740
LOGICAL RST,DEBUG                                              GR018750
DATA SIGN /-1.0D/                                               GR018760
C THE FOLLOWING DATA ARE FOR PERKIN-ELMER COMPUTERS             GR018770
DATA EPMACH,UFLOW/Z341000000000000,Z001000000000000/              GR018780
C THE FOLLOWING DATA ARE FOR VAX COMPUTERS                      GR018790
DATA EPMACH,UFLOW/Z0000250000000000,Z000008000000000/              GR018800
C                                                               GR018810
C EPMACH = Y'3410000000000000'                                     GR018820
C UFLOW = Y'00100000'                                             GR018830
C EPMACH = DBLE(16**(-13))                                         GR018840
C UFLOW = DBLE(16**(-65))                                         GR018850
IF(A.EQ.B) THEN                                                 GR018860
  R=0.                                                       GR018870
  E=0.                                                       GR018880
  NINT=0                                                 GR018890
  IFLAG=0                                              GR018900
  KF=1                                                 GR018910
  FMIN=F(A)                                            GR018920
  FMAX=FMIN                                           GR018930
  GO TO 20                                              GR018940
ENDIF                                                       GR018950
IF(RST) THEN                                                 GR018960
  IF(IFLAG.LT.3) GO TO 15                                GR018970
  GO TO 20                                              GR018980
ENDIF                                                       GR018990
KF=0                                                       GR019000
IF(EPS .LE. 0. .OR. NMAX .LE. 1 .OR. NINT .LE. 0) THEN          GR019010
  IFLAG=7                                              GR019020
  GO TO 20                                              GR019030
ENDIF                                                       GR019040
IF(NINT.EQ.1) THEN                                         GR019050
C 1 THEN                                                 GR019060
  W(1,1)=A                                              GR019070
  W(2,2)=B                                              GR019080
  W(1,5)=A                                              GR019090
  W(1,6)=B                                              GR019100
  W(2,5)=A                                              GR019110
  W(2,6)=B                                              GR019120
  W(1,2)=A+(B-A)/2.0D0                               GR019130
  W(2,1)=A+(B-A)/2.0D0                               GR019140
  NINT=2                                              GR019150
ELSE                                                       GR019160
  IF(W(1,1).NE.A .AND. W(NINT+1,1).NE.B) THEN            GR019170
    IFLAG=8                                              GR019180
    GO TO 20                                              GR019190
  ENDIF                                                       GR019200
  W(1,5)=A                                              GR019210
  DO 89 I=1,NINT                                         GR019220
    W(I,2)=W(I+1,1)                                       GR019230
    W(I,5)=W(I,1)                                       GR019240

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        W(I,6)=W(I,2)                               GR019250
89      CONTINUE                                GR019260
      ENDIF                                     GR019270
C
      DEBUG=.FALSE.                             GR019280
      IFLAG = 0                                 GR019290
      IROFF=0                                  GR019300
      RABS=0.0                                 GR019310
      DO 3 I=1,NINT                            GR019320
        CALL GL15T(F,W(I,1),W(I,2),W(I,5),W(I,6),   GR019340
1          W(I,4),W(I,3),RAB,RAV,FMN,FMX)       GR019350
        IF(DEBUG) WRITE(7 ,*)'INITIALIZE',(W(I,J),J=1,6) GR019360
        KF=KF+15                                GR019370
        IF(I.EQ.1)                                GR019380
1      THEN                                     GR019390
          R=W(I,4)                                GR019400
          E=W(I,3)                                GR019410
          RABS=RABS+RAB                           GR019420
          FMIN=FMN                                GR019430
          FMAX=FMX                                GR019440
        ELSE                                     GR019450
          R=R+W(I,4)                                GR019460
          E=E+W(I,3)                                GR019470
          RABS=RABS+RAB                           GR019480
          FMAX=DMAX1(FMAX,FMX)                   GR019490
          FMIN=DMIN1(FMIN,FMN)                   GR019500
        ENDIF                                    GR019510
3  CONTINUE                                 GR019520
      DO 10 I=NINT+1,NMAX                      GR019530
        W(I,3) = 0.                                GR019540
10 CONTINUE                                GR019550
15 CONTINUE                                GR019560
C
C  MAIN SUBPROGRAM LOOP                     GR019570
C
      IF(100.D0*EPMACH*RABS.GE.DABS(R) .AND. E.LT.EPS)GO TO 20  GR019600
      EB=DMAX1(100.D0*UFLOW,DMAX1(EPS,50.D0*EPMACH)*DABS(R))  GR019610
      IF(E.LE.EB) GO TO 20                         GR019620
      IF (NINT.LT.NMAX)                            GR019630
1 THEN                                     GR019640
        NINT = NINT+1                            GR019650
        C = NINT                                GR019660
      ELSE                                     GR019670
        C=0                                      GR019680
16  IF(C.EQ.NMAX) THEN                     GR019690
        IFLAG=5                                GR019700
        GO TO 20                                GR019710
      ENDIF                                    GR019720
      C=C+1                                    GR019730
      IF(W(C,3).GT.0.D0) GO TO 16             GR019740
    END IF                                    GR019750
C
      LOC=ISAMAX(NINT,W(1,3),1)                GR019760
      CALL LSE(W,NMAX,NINT,LOC)                 GR019770
      IF(DEBUG)WRITE(7,200)LOC,(W(LOC,I),I=1,6),R,E   GR019780
      XM = W(LOC,1)+(W(LOC,2)-W(LOC,1))/2.           GR019790

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IF ((DABS(W(LOC,1)),DABS(W(LOC,2))).GT. GR019800
1   ((1.D0+100.D0*EPMACH)*(DABS(XM)+0.1D+04*UFLW))) GR019810
2   THEN GR019820
    CALL GL15T(F,W(LOC,1),XM,W(LOC,5),W(LOC,6), GR019830
1           TR1,TE1,RAB,RAV,FMINL,FMAXL) GR019840
    KF=KF+15 GR019850
    IF (TE1.LT.(EB*(XM-W(LOC,1))/(B-A))) GR019860
A       TE1 = TE1*SIGN GR019870
    CALL GL15T(F,XM,W(LOC,2),W(LOC,5),W(LOC,6), GR019880
1           TR2,TE2,RAB,RAV,FMINR,FMAXR) GR019890
    KF=KF+15 GR019900
    FMIN=DMIN1(FMIN,FMINL,FMINR) GR019910
    FMAX=DMAX1(FMAX,FMAXL,FMAXR) GR019920
    IF (TE2.LT.(EB*(W(LOC,2)-XM)/(B-A))) TE2=TE2*SIGN GR019930
    TE = DABS(W(LOC,3)) GR019940
    TR = W(LOC,4) GR019950
    W(C,3) = TE2 GR019960
    W(C,4) = TR2 GR019970
    W(C,1) = XM GR019980
    W(C,2) = W(LOC,2) GR019990
    W(C,5) = W(LOC,5) GR020000
    W(C,6) = W(LOC,6) GR020010
    W(LOC,3) = TE1 GR020020
    W(LOC,4) = TR1 GR020030
    W(LOC,2) = XM GR020040
    IF(DEBUG)WRITE(7,200)C,(W(C,K),K=1,6) GR020050
    IF(DEBUG)WRITE(7,200)LOC,(W(LOC,K),K=1,6) GR020060
    E = E-TE+(DABS(TE1)+DABS(TE2)) GR020070
    R = R-TR+(TR1+TR2) GR020080
    IF(DEBUG)WRITE(7,*)NINT,R,E GR020090
    IF(DABS(DABS(TE1)+DABS(TE2)-TE).LT..001D0*TE) THEN GR020100
        IROFF=IROFF+1 GR020110
        IF(IROFF.GE.10) THEN GR020120
            IFLAG=4 GR020130
            GO TO 20 GR020140
        ENDIF GR020150
    ENDIF GR020160
    ELSE GR020170
        IF (EB.GT.W(LOC,3)) GR020180
1        THEN GR020190
            W(LOC,3) = 0. GR020200
        ELSE GR020210
            IFLAG=6 GR020220
            GO TO 20 GR020230
        END IF GR020240
    END IF GR020250
    GO TO 15 GR020260
C
C      ALL EXITS FROM HERE GR020270
C
20 CONTINUE GR020280
    IF(IFLAG.GE.4)RETURN GR020290
    IFLAG=3 GR020310
    T=EPS*DABS(R) GR020320
    IF(E.GT.EPS .AND. E.GT.T)RETURN GR020330
    GR020340

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IFLAG=2 GR020350
IF(E.GT.EPS .AND. E.LT.T)RETURN GR020360
IFLAG=1 GR020370
IF(E.LT.EPS .AND. E.GT.T)RETURN GR020380
IFLAG=0 GR020390
RETURN GR020400
200 FORMAT (I4,8(E11.3)) GR020410
END GR020420
C*****
C
SUBROUTINE LSE (WORK,NMAX,NI,LOC) GR020430
C
C*****
C THIS SUBPROGRAM FINDS THE CELL IN THE WORK AREA GR020470
C THAT HAS THE LARGEST ABSOLUTE ERROR AND GR020480
C RETURNS THE LOCATION OF THAT CELL. GR020490
C
C DOUBLE PRECISION ERROR,WORK GR020520
DIMENSION WORK(NMAX,6) GR020540
C
C INITIALIZE VARIABLES GR020550
C
C ERROR = DABS(WORK(1,3)) GR020560
LOC = 1 GR020570
C
C MAIN SUBPROGRAM LOOP GR020600
C
DO 20 I=1,NI GR020630
IF (DABS(WORK(I,3)).GT.ERROR) THEN GR020640
ERROR = DABS(WORK(I,3)) GR020650
LOC = I GR020660
END IF GR020670
20 CONTINUE GR020680
RETURN GR020690
END GR020700
C*****
C
SUBROUTINE GL15T(F,A,B,XL,XR,R,AE,RA, GR020730
1 RASC,FMIN,FMAX) GR020740
C
C*****
C***AUTHORS ROBERT PIESSENS AND ELISE DE DONCKER GR020780
C APPL. MATH. AND PROGR. DIV.- K.U.LEUVEN GR020790
C DAVID KAHANER, NBS WASHINGTON GR020800
C ..... GR020810
C PURPOSE GR020820
C TO COMPUTE I = INTEGRAL OF G(X) OVER (A,B), GR020830
C WITH ERROR ESTIMATE GR020840
C J = INTEGRAL OF DABS(G) OVER (A,B) GR020850
C
C PARAMETERS GR020860
C ON ENTRY GR020880
C F - DOUBLE PRECISION GR020890

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C FUNCTION SUBPROGRAM DEFINING THE INTEGRAND GR020900  
 C FUNCTION F(X). THE ACTUAL NAME FOR F NEEDS GR020910  
 C TO BE DECLARED E X T E R N A L IN THE GR020920  
 C CALLING PROGRAM. GR020930  
 C THE FUNCTION G(X) IS DEFINED TO BE GR020940  
 C G(X)=F(PHI(X))\*PHIP(X) GR020950  
 C WHERE PHI(X) IS THE CUBIC GIVEN BY GR020960  
 C THE ARITHMETIC STATEMENT FUNCTION BELOW. GR020970  
 C PHIP(X) IS ITS DERIVATIVE. THE VARIABLES GR020980  
 C XL AND XR ARE THE LEFT AND RIGHT ENDPOINTS GR020990  
 C OF A PARENT INTERVAL OF WHICH (A,B) IS A PART. GR021000  
 C GR021010  
 C A - DOUBLE PRECISION GR021020  
 C LOWER LIMIT OF INTEGRATION GR021030  
 C GR021040  
 C B - DOUBLE PRECISION GR021050  
 C UPINTG LIMIT OF INTEGRATION GR021060  
 C GR021070  
 C XL - DOUBLE PRECISION GR021080  
 C LEFT ENDPOINT OF PARENT INTERVAL GR021090  
 C GR021100  
 C XR - DOUBLE PRECISION GR021110  
 C RIGHT ENDPOINT OF PARENT INTERVAL GR021120  
 C GR021130  
 C ON RETURN GR021140  
 C R - DOUBLE PRECISION GR021150  
 C APPROXIMATION TO THE INTEGRAL I GR021160  
 C R IS COMPUTED BY APPLYING THE 15-POINT GR021170  
 C KRONROD RULE (RESK) OBTAINED BY OPTIMAL GR021180  
 C ADDITION OF ABSCESSAE TO THE 7-POINT GAUSS GR021190  
 C RULE (RESG). GR021200  
 C GR021210  
 C AE - DOUBLE PRECISION GR021220  
 C ESTIMATE OF THE MODULUS OF THE ABSOLUTE ERROR, GR021230  
 C WHICH SHOULD NOT EXCEED DABS(I-R) GR021240  
 C GR021250  
 C RA - DOUBLE PRECISION GR021260  
 C APPROXIMATION TO THE INTEGRAL J GR021270  
 C GR021280  
 C RASC - DOUBLE PRECISION GR021290  
 C APPROXIMATION TO THE INTEGRAL OF DABS(G-I/(B-A)) GR021300  
 C OVER (A,B) GR021310  
 C GR021320  
 C FMAX, FMIN - DOUBLE PRECISION GR021330  
 C MAX AND MIN VALUES OF THE FUNCTION F ON (A,B) GR021340  
 C SUBROUTINES OR FUNCTIONS NEEDED GR021350  
 C - F (USER-PROVIDED FUNCTION) GR021360  
 C - R1MACH GR021370  
 C - FORTRAN DABS, DMAX1, DMIN1 GR021380  
 C GR021390  
 C ..... GR021400  
 C \*\*\*END PROLOGUE GR021410  
 C GR021420  
 C DOUBLE PRECISION A,AE,B,DHLGTH,EPMACH,F,FC,FSUM,FVAL1,FVAL2, GR021430  
 \* FV1,FV2,HLGTH,RA,RASC,RESG,RESKH,R,R1MACH,UFLOW, GR021440

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* WG,WGK,XGK                               GR021450
DOUBLE PRECISION XL,XR,CENTR,ABSC,U,FMAX,FMIN,PHI,PHIP   GR021460
INTEGER J,JTW,JTWM1                           GR021470
C                                         GR021480
DIMENSION FV1(7),FV2(7),WG(4),WGK(8),XGK(8)           GR021490
C                                         GR021500
C THE ABSCISSAE AND WEIGHTS ARE GIVEN FOR THE INTERVAL (-1,1) GR021510
C BECAUSE OF SYMMETRY ONLY THE POSITIVE ABSCISSAE AND THEIR    GR021520
C CORRESPONDING WEIGHTS ARE GIVEN.                      GR021530
C                                         GR021540
C XGK - ABSCISSAE OF THE 15-POINT KRONROD RULE          GR021550
C XGK(2), XGK(4), ... ABSCISSAE OF THE 7-POINT          GR021560
C GAUSS RULE                                         GR021570
C XGK(1), XGK(3), ... ABSCISSAE WHICH ARE OPTIMALLY GR021580
C ADDED TO THE 7-POINT GAUSS RULE                   GR021590
C                                         GR021600
C WGK - WEIGHTS OF THE 15-POINT KRONROD RULE          GR021610
C                                         GR021620
C WG - WEIGHTS OF THE 7-POINT GAUSS RULE             GR021630
C                                         GR021640
C DATA XGK(1),XGK(2),XGK(3),XGK(4),XGK(5),XGK(6),XGK(7),XGK(8)/  GR021650
*      0.9914553711208126D+00,     0.9491079123427585D+00,  GR021660
*      0.8648644233597691D+00,     0.7415311855993944D+00,  GR021670
*      0.5860872354676911D+00,     0.4058451513773972D+00,  GR021680
*      0.2077849550078985D+00,     0.0D+00                      /  GR021690
DATA WGK(1),WGK(2),WGK(3),WGK(4),WGK(5),WGK(6),WGK(7),WGK(8)/  GR021700
*      0.2293532201052922D-01,     0.6309209262997855D-01,  GR021710
*      0.1047900103222502D+00,     0.1406532597155259D+00,  GR021720
*      0.1690047266392679D+00,     0.1903505780647854D+00,  GR021730
*      0.2044329400752989D+00,     0.2094821410847278D+00/  GR021740
DATA WG(1),WG(2),WG(3),WG(4)/                         GR021750
*      0.1294849661688697D+00,     0.2797053914892767D+00,  GR021760
*      0.3818300505051189D+00,     0.4179591836734694D+00/  GR021770
DATA EPMACH,UFLOW/Z341000000000000,Z0010000000000000/  GR021780
C                                         GR021790
PHI(U)=XR-(XR-XL)*U*U*(2.D0*U+3.D0)                  GR021800
PHIP(U)=-6.D0*U*(U+1.D0)                            GR021810
C                                         GR021820
C LIST OF MAJOR VARIABLES                         GR021830
C -----
C CENTR - MID POINT OF THE INTERVAL            GR021840
C HLGTH - HALF-LENGTH OF THE INTERVAL          GR021850
C ABSC - ABSCISSA                             GR021860
C FVAL* - FUNCTION VALUE                      GR021870
C RESG - R OF THE 7-POINT GAUSS FORMULA       GR021880
C RESK - R OF THE 15-POINT KRONROD FORMULA     GR021890
C RESKH - APPROXIMATION TO THE MEAN VALUE OF F OVER (A,B),
C          I.E. TO I/(B-A)                      GR021900
C                                         GR021910
C MACHINE DEPENDENT CONSTANTS                 GR021920
C -----
C EPMACH IS THE LARGEST RELATIVE SPACING.        GR021930
C UFLOW IS THE SMALLEST POSITIVE MAGNITUDE.       GR021940
C                                         GR021950
C                                         GR021960
C                                         GR021970
C                                         GR021980
C                                         GR021990

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C                                         GR022000
C***FIRST EXECUTABLE STATEMENT           GR022010
C   EPMACH = DBLE(16**(-13))            GR022020
C   UFLOW = DBLE(16**(-65))             GR022030
C
C   HLGTH = 0.5D+00*(B-A)                GR022040
C   CENTR = A+HLGTH                      GR022050
C   DHLGTH = DABS(HLGTH)                 GR022060
C
C   COMPUTE THE 15-POINT KRONROD APPROXIMATION TO      GR022080
C   THE INTEGRAL, AND ESTIMATE THE ABSOLUTE ERROR.       GR022090
C
C   U=(CENTR-XR)/(XR-XL)                  GR022100
C   FMIN=F(PHI(U))                      GR022110
C   FMAX=FMIN
C   FC=FMIN*PHIP(U)                     GR022120
C   RESG = FC*WG(4)                      GR022130
C   RESK = FC*WGK(8)                     GR022140
C   RA = DABS(RESK)                     GR022150
C   DO 10 J=1,3                         GR022160
C     JTW = J*2                           GR022170
C     ABSC = HLGTH*XGK(JTW)              GR022180
C     U=(CENTR-ABSC-XR)/(XR-XL)          GR022190
C     FVAL1 = F(PHI(U))                  GR022200
C     FMAX=DMAX1(FMAX,FVAL1)            GR022210
C     FMIN=DMIN1(FMIN,FVAL1)            GR022220
C     FVAL1=FVAL1*PHIP(U)                GR022230
C     U=(CENTR+ABSC-XR)/(XR-XL)          GR022240
C     FVAL2=F(PHI(U))                  GR022250
C     FMAX=DMAX1(FMAX,FVAL2)            GR022260
C     FMIN=DMIN1(FMIN,FVAL2)            GR022270
C     FVAL2=FVAL2*PHIP(U)                GR022280
C     FV1(JTW) = FVAL1                  GR022290
C     FV2(JTW) = FVAL2                  GR022300
C     FSUM = FVAL1+FVAL2                GR022310
C     RESG = RESG+WG(J)*FSUM            GR022320
C     RESK = RESK+WGK(JTW)*FSUM         GR022330
C     RA = RA+WGK(JTW)*(DABS(FVAL1)+DABS(FVAL2))    GR022340
10 CONTINUE                                GR022350
DO 15 J = 1,4                            GR022360
  JTWM1 = J*2-1                          GR022370
  ABSC = HLGTH*XGK(JTWM1)                GR022380
  U=(CENTR-ABSC-XR)/(XR-XL)              GR022390
  FVAL1=F(PHI(U))                      GR022400
  FMAX=DMAX1(FMAX,FVAL1)                GR022410
  FMIN=DMIN1(FMIN,FVAL1)                GR022420
  FVAL1=FVAL1*PHIP(U)                  GR022430
  U=(CENTR+ABSC-XR)/(XR-XL)              GR022440
  FVAL2=F(PHI(U))                      GR022450
  FMAX=DMAX1(FMAX,FVAL2)                GR022460
  FMIN=DMIN1(FMIN,FVAL2)                GR022470
  FVAL2=FVAL2*PHIP(U)                  GR022480
  FV1(JTWM1) = FVAL1                  GR022490
  FV2(JTWM1) = FVAL2                  GR022500
  FSUM = FVAL1+FVAL2                  GR022510

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      RESK = RESK+WGK(JTWM1)*FSUM           GR022550
      RA = RA+WGK(JTWM1)*(DABS(FVAL1)+DABS(FVAL2))   GR022560
15 CONTINUE                                GR022570
      RESKH = RESK*0.5D+00                  GR022580
      RASC = WGK(8)*DABS(FC-RESKH)        GR022590
      DO 20 J=1,7                         GR022600
      RASC = RASC+WGK(J)*(DABS(FV1(J)-RESKH)+DABS(FV2(J)-RESKH))   GR022610
20 CONTINUE                                GR022620
      R = RESK*HLGTH                      GR022630
      RA = RA*DHLGTH                      GR022640
      RASC = RASC*DHLGTH                  GR022650
      AE = DABS((RESK-RESG)*HLGTH)       GR022660
      IF(RASC.NE.0.0D+00.AND.AE.NE.0.0D+00)   GR022670
      * AE = RASC*DMIN1(0.1D+01,             GR022680
      * (0.2D+03*AE/RASC)**1.5D+00)         GR022690
      IF(RA.GT.UFLOW/(0.5D+02*EPMACH)) AE = DMAX1   GR022700
      * ((EPMACH*0.5D+02)*RA,AE)           GR022710
      RETURN                                GR022720
      END                                    GR022730
*****
C*****                                         GR022740
C                                         GR022750
      SUBROUTINE QUADRT(A,ROOT1,ROOT2,TOL,NOROOT)   GR022760
C                                         GR022770
*****
C                                         GR022780
C                                         GR022790
C      TO SOLVE ANY QUADRATIC EQUATION OF THE FORM   GR022800
C                                         GR022810
C      A(1)*X**2+A(2)*X+A(3) = 0                 GR022820
C                                         GR022830
C      DOUBLE PRECISION A,ROOT1,ROOT2,X,Y,Z,W,ZERO   GR022840
      DIMENSION A(3),ROOT1(2),ROOT2(2)               GR022850
C                                         GR022860
      ZERO = TOL/10.0                               GR022870
C                                         GR022880
      IF (DABS(A(1))-ZERO)>2,2,1                 GR022890
2     IF (DABS(A(2))-ZERO) 3,3,4                GR022900
3     NOROOT =0                                 GR022910
      RETURN                                GR022920
C                                         GR022930
1     NOROOT= 2                                GR022940
      X = A(2)*A(2) - 4.0*A(1)*A(3)            GR022950
      Y = A(1) + A(1)                          GR022960
      Z = DSQRT(DABS(X))/Y                     GR022970
      W = -A(2)/Y                            GR022980
C                                         GR022990
      IF (X.LT.0.0) GO TO 7                   GR023000
      ROOT1(1)=W+Z                           GR023010
      ROOT1(2)=0.0                           GR023020
      ROOT2(1)=W-Z                           GR023030
      ROOT2(2)=0.0                           GR023040
      RETURN                                GR023050
C                                         GR023060
7     ROOT1(1)=W                           GR023070
      ROOT1(2)=Z                           GR023080
      ROOT2(1)=W                           GR023090

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ROOT2(2)=-Z                               GR023100
RETURN                                     GR023110
C                                         GR023120
4 NOROOT=1                                GR023130
  ROOT1(1) = -A(3)/A(2)                   GR023140
  ROOT1(2) = 0.0                           GR023150
RETURN                                     GR023160
END                                       GR023170
*****
C                                         GR023180
C                                         GR023190
SUBROUTINE DCRTNI(X,F,DERF,XST,EPS,IEND,IER) GR023200
C                                         GR023210
*****
C                                         GR023220
C                                         GR023230
C PURPOSE                                    GR023240
TO SOLVE PHI = 0                           GR023250
BY MEANS OF NEWTON-S ITERATION METHOD.    GR023260
C                                         GR023270
C USAGE                                     GR023280
CALL DCRTNI (X,F,DERF,XST,EPS,IEND,IER)   GR023290
C                                         GR023300
C DESCRIPTION OF PARAMETERS                 GR023310
X      - DOUBLE COMPLEX RESULTANT ROOT OF EQUATION F(X)=0. GR023320
F      - DOUBLE COMPLEX RESULTANT FUNCTION VALUE AT      GR023330
      ROOT X.                                         GR023340
DERF   - DOUBLE COMPLEX RESULTANT VALUE OF DERIVATIVE   GR023350
      AT ROOT X.                                         GR023360
XST    - DOUBLE COMPLEX INPUT VALUE WHICH SPECIFIES THE GR023370
      INITIAL GUESS OF THE ROOT X.                         GR023380
EPS    - SINGLE PRECISION INPUT VALUE WHICH SPECIFIES THE GR023390
      UPPER BOUND OF THE ERROR OF RESULT X.                GR023400
IEND   - MAXIMUM NUMBER OF ITERATION STEPS SPECIFIED.  GR023410
IER    - RESULTANT ERROR PARAMETER CODED AS FOLLOWS     GR023420
      IER=0 - NO ERROR,                                 GR023430
      IER=1 - NO CONVERGENCE AFTER IEND ITERATION STEPS, GR023440
      IER=2 - AT ANY ITERATION STEP DERIVATIVE DERF WAS GR023450
      EQUAL TO ZERO.                                  GR023460
C                                         GR023470
C                                         GR023480
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED GR023490
PHIO,AND DPHIO                                GR023500
C                                         GR023510
C METHOD                                     GR023520
SOLUTION OF EQUATION F(X)=0 IS DONE BY MEANS OF NEWTON-S GR023530
ITERATION METHOD, WHICH STARTS AT THE INITIAL GUESS XST OF GR023540
A ROOT X. CONVERGENCE IS QUADRATIC IF THE DERIVATIVE OF   GR023550
F(X) AT ROOT X IS NOT EQUAL TO ZERO. ONE ITERATION STEP   GR023560
REQUIRES ONE EVALUATION OF F(X) AND ONE EVALUATION OF THE   GR023570
DERIVATIVE OF F(X).                                     GR023580
FOR REFERENCE, SEE R. ZURMUEHL, PRAKТИSCHE MATHEMATIK FUER   GR023590
INGENIEURE UND PHYSIKER, SPRINGER, BERLIN/GÖTTINGEN/   GR023600
HEIDELBERG, 1963, PP.12-17.                          GR023610
C                                         GR023620
C                                         GR023630
DOUBLE COMPLEX X,F,DERF,XST,TOL,DX,PHIO,DPHIO          GR023640

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DOUBLE PRECISION A,DTOL,DTOLF           GR023650
EXTERNAL PHIO,DPHIO                      GR023660
C   PREPARE ITERATION                     GR023670
    IER=0                                GR023680
    X=XST                               GR023690
    TOL=X                               GR023700
    F = PHIO(TOL)                        GR023710
    DERF = DPHIO(TOL)                     GR023720
    DTOLF=100.D0*EPS                      GR023730
C                                         GR023740
C                                         GR023750
C   START ITERATION LOOP                 GR023760
    DO 6 I=1,IEND                       GR023770
    IF(CDABS(F))1,7,1                   GR023780
C                                         GR023790
C   EQUATION IS NOT SATISFIED BY X      GR023800
    1 IF(CDABS(DERF))2,8,2              GR023810
C                                         GR023820
C   ITERATION IS POSSIBLE                GR023830
    2 DX=F/DERF                         GR023840
    X=X-DX                            GR023850
    TOL=X                             GR023860
    F = PHIO(TOL)                        GR023870
    DERF = DPHIO(TOL)                     GR023880
C                                         GR023890
C   TEST ON SATISFACTORY ACCURACY       GR023900
    DTOL=DBLE(EPS)                      GR023910
    A=CDABS(X)                          GR023920
    IF(A-1.D0)4,4,3                     GR023930
    3 DTOL=DTOL*A                      GR023940
    4 IF(CDABS(DX)-DTOL)5,5,6          GR023950
    5 IF(CDABS(F)-DTOLF)7,7,6          GR023960
    6 CONTINUE                           GR023970
C   END OF ITERATION LOOP               GR023980
C                                         GR023990
C                                         GR024000
C   NO CONVERGENCE AFTER IEND ITERATION STEPS. ERROR RETURN. GR024010
    IER=1                                GR024020
    7 RETURN                             GR024030
C                                         GR024040
C   ERROR RETURN IN CASE OF ZERO DIVISOR GR024050
    8 IER=2                                GR024060
    RETURN                                GR024070
    END                                   GR024080
C*****                                         GR024090
C                                         GR024100
C   SUBROUTINE RSORT(A,B,R,N,M,MS)        GR024110
C                                         GR024120
C*****                                         GR024130
    DIMENSION A(1),B(1),R(1)             GR024140
C                                         GR024150
C   MOVE SORTING KEY VECTOR TO FIRST COLUMN OF OUTPUT MATRIX GR024160
C   AND BUILD ORIGINAL SEQUENCE LIST IN SECOND COLUMN          GR024170
C                                         GR024180
C   DO 10 I=1,N                           GR024190

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R(I)=B(I)                               GR024200
I2=I+N                                 GR024210
10 R(I2)=I                               GR024220
C                                         GR024230
C     SORT ELEMENTS IN SORTING KEY VECTOR (ORIGINAL SEQUENCE LIST   GR024240
C     IS RESEQUENCED ACCORDINGLY)                                     GR024250
C                                         GR024260
L=N+1                                    GR024270
20 ISORT=0                                GR024280
L=L-1                                    GR024290
IF(L .LT. 2) GO TO 50                  GR024300
DO 40 I=2,L                             GR024310
IF(R(I)-R(I-1)) 30,40,40               GR024320
30 ISORT=1                                GR024330
RSAVE=R(I)                               GR024340
R(I)=R(I-1)                               GR024350
R(I-1)=RSAVE                            GR024360
I2=I+N                                 GR024370
SAVER=R(I2)                             GR024380
R(I2)=R(I2-1)                           GR024390
R(I2-1)=SAVER                           GR024400
40 CONTINUE                               GR024410
IF(ISORT) 20,50,20                      GR024420
C                                         GR024430
C     MOVE ROWS FROM MATRIX A TO MATRIX R (NUMBER IN SECOND COLUMN  GR024440
C     OF R REPRESENTS ROW NUMBER OF MATRIX A TO BE MOVED)           GR024450
C                                         GR024460
50 DO 80 I=1,N                           GR024470
C                                         GR024480
C     GET ROW NUMBER IN MATRIX A                         GR024490
C                                         GR024500
I2=I+N                                 GR024510
IN=R(I2)                                GR024520
C                                         GR024530
IR=I-N                                 GR024540
DO 80 J=1,M                             GR024550
C                                         GR024560
C     LOCATE ELEMENT IN OUTPUT MATRIX                     GR024570
C                                         GR024580
IR=IR+N                                GR024590
C                                         GR024600
C     LOCATE ELEMENT IN INPUT MATRIX                     GR024610
C                                         GR024620
CALL LOC(IN,J,IA,N,M,MS)                GR024630
C                                         GR024640
C     TEST FOR ZERO ELEMENT IN DIAGONAL MATRIX          GR024650
C                                         GR024660
IF(IA) 60,70,60                          GR024670
C                                         GR024680
C     MOVE ELEMENT TO OUTPUT MATRIX                     GR024690
C                                         GR024700
60 R(IR)=A(IA)                           GR024710
GO TO 80                                GR024720
70 R(IR)=0                               GR024730
80 CONTINUE                               GR024740

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RETURN                               GR024750
END                                 GR024760
C*****SUBROUTINE LOC(I,J,IR,N,M,MS)***** GR024770
C                                         GR024780
C                                         GR024790
C                                         GR024800
C*****GR024810
C                                         GR024820
C                                         GR024830
C PURPOSE                           GR024840
C COMPUTE A VECTOR SUBSCRIPT FOR AN ELEMENT IN A MATRIX OF
C SPECIFIED STORAGE MODE           GR024850
C                                         GR024860
C USAGE                             GR024870
C CALL LOC (I,J,IR,N,M,MS)          GR024880
C                                         GR024890
C DESCRIPTION OF PARAMETERS         GR024900
C I - ROW NUMBER OF ELEMENT        GR024910
C J - COLUMN NUMBER OF ELEMENT     GR024920
C IR - RESULTANT VECTOR SUBSCRIPT GR024930
C N - NUMBER OF ROWS IN MATRIX    GR024940
C M - NUMBER OF COLUMNS IN MATRIX GR024950
C MS - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX   GR024960
C          0 - GENERAL             GR024970
C          1 - SYMMETRIC            GR024980
C          2 - DIAGONAL             GR024990
C                                         GR025000
C REMARKS                           GR025010
C NONE                              GR025020
C M IS UNUSED BUT IS REQUIRED TO BE PRESENT   GR025030
C                                         GR025040
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED   GR025050
C NONE                                GR025060
C                                         GR025070
C METHOD                            GR025080
C MS=0     SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*M ELEMENTS GR025090
C          IN STORAGE (GENERAL MATRIX)                      GR025100
C MS=1     SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*(N+1)/2 IN GR025110
C          STORAGE (UPPER TRIANGLE OF SYMMETRIC MATRIX). IF   GR025120
C          ELEMENT IS IN LOWER TRIANGULAR PORTION, SUBSCRIPT IS GR025130
C          CORRESPONDING ELEMENT IN UPPER TRIANGLE.           GR025140
C MS=2     SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N ELEMENTS  GR025150
C          IN STORAGE (DIAGONAL ELEMENTS OF DIAGONAL MATRIX). GR025160
C          IF ELEMENT IS NOT ON DIAGONAL (AND THEREFORE NOT IN GR025170
C          STORAGE), IR IS SET TO ZERO.                     GR025180
C                                         GR025190
C                                         GR025200
C                                         GR025210
C IX=I                               GR025220
C JX=J                               GR025230
C IF(MS-1) 10,20,30                  GR025240
10 IRX=N*(JX-1)+IX                 GR025250
GO TO 36                            GR025260
20 IF(IX-JX) 22,24,24                GR025270
22 IRX=IX+(JX*JX-JX)/2              GR025280
GO TO 36                            GR025290

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|                        |          |
|------------------------|----------|
| 24 IRX=JX+(IX*IX-IX)/2 | GR025300 |
| GO TO 36               | GR025310 |
| 30 IRX=0               | GR025320 |
| IF(IX-JX) 36,32,36     | GR025330 |
| 32 IRX=IX              | GR025340 |
| 36 IR=IRX              | GR025350 |
| RETURN                 | GR025360 |
| END                    | GR025370 |



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 Document describes a computer program; SF-185, FIPS Software Summary, is attached.

## 11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

This report is a FORTRAN program to compute the Green's functions of an infinite plate. The Green's function,  $G_{ij}(\xi, x, t)$ , is defined as the  $i$ th component of the displacement at  $x$  due to a point force of step-function time dependency acting at  $\xi$  in the  $j$ th direction initiated at  $t=0$ . The Green's function is the fundamental solution of the transient elastic wave propagation problem. In general, the displacement field  $u(\xi, x, t)$  at  $x$  due to a point force of arbitrary time dependence acting at  $\xi$  can be computed by a convolution integration; i.e.,

$$u_i(\xi, x, t) = \int_0^{\infty} G_{ij}(\xi, x, \tau) f_j(\tau-t) d\tau. \quad (1)$$

Here,  $G_{ij}$  is the time derivative of  $G_{ij}$  and  $f_j(t)$  is the point force component of arbitrary time dependence acting in the  $j$ th direction (summation over repeated indices is used). Displacement due to point dipoles or couple forces can be represented by the spatial derivatives of  $G_{ij}$ . Displacement produced by a dynamic force distributed over a finite area can also be computed by numerical integration using the Green's function as the kernel of the integral over the finite area.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)  
Acoustic emission; dynamic displacement due to point impact; Green's function; ray theory; transient wave propagation; waveform analysis

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