Volume 5 • Out. 2017
ISSN 2317-126X • US$: 15.00

American Journal of Hydropower, Water and Environment Systems

Technical Papers

IAHR AIRH

Published with the support of
Hydraulic Machinery and Systems
International Association

latinamerican WORKING GROUP

American Journal of Hydropower, Water and Environment Systems

Editorial

We now come to the AJHPWS´s fifth edition.

This publication is one of the actions undertaken by the Latin American Working Group of IAHR´s Hydraulic Machines and Systems Committee.

The fight in searching new papers and prestige from scientific community is incessant. An effective contribution for sustainable development is one of our purposes.

All our researches have the potential to promote scientific knowledge and to support new policies aiming the development of new technologies from the technical, social and economic point of view, in addition to subsidize sustainable development and the quest for a better future.

Thus, dissemination of knowledge, methodologies and procedures of physical phenomena scientific modeling and solutions to reach technological development is one of the groups´ goals and consequently, of this journal.

In this case, this journal gradually becomes an eclectic vehicle within the limits of water resources and environment, if there are, and in addition it is interesting for the academy, considering that most part of papers come from research groups of several universities and research centers distributed in Brazil and Latin America.

This edition addresses several subjects such as: the project and building of a small hydropower plant aiming to become a learning tool for teaching and training of engineering students; suitability of an existing reservoir to operate as a mini hydropower plant, according to regulatory environment of not centralized micro generation in Brazil; comparative studies of methodologies for hydraulics transient calculation and finally, studies addressing management issues of water resources.

We do hope to contribute in any way in this direction. We look forward you to appreciate reading of selected papers for this edition of AJHWES.

Yours faithfully,
Geraldo Lucio Tiago Filho
Editor in Chief

Regina Mambeli Barros
Technical Editor
American Journal of Hydropower, Water and Environment Systems

IAHR AIRH

IAHR DIVISION I: HYDRAULICS
TECHNICAL COMMITTEE: HYDRAULIC
MACHINERY AND SYSTEMS

Federal University of Itajubá - UNIFEI
Av. BPS, 1303 - Bairro Pinheirinho
Itajubá - MG - Brasil - CEP: 37500-903

Catalographic card prepared by Mauá Library – Librarian Margareth Ribeiro – CRB_6/1700

R454

American Journal of Hydropower, Water and Environment Systems, LAWG-IAHR, v.1, 2014 – Itajubá: LAWG-IAHR, 2015 - v.5, out. 2017.

Trimestral,
Editors Chief: Geraldo Lúcio Tiago Filho/Eduard Egusquiza
Journalist in Charge: Adriana Barbosa – MTb_MG 05984
ISSN 2317-126X

1. Hydropower. 2. Water. 3. Enviroment Systems.
II. Latin American Working Group

LAWG-IAHR Executive Secretariat
Lucia Garrido Rios
latiniahrsecex@gmail.com

ISSN 2317-126X

9 772317 126001 05

American Journal of Hydropower, Water and Environment Systems

A publication of Latin American Working Group of the International Association for Hydro-Environment Engineering and Research-IAHR

All papers must be submitted in English. In case the author wants to translate the article through the journal all costs for the translation will be charged on the account of the author.

1. Formatting articles

1.1. Article structure

1.1.1 Subdivision - numbered sections

Divide your article into clearly defined and numbered sections. Subsections should be numbered 1.1 (then 1.1.1, 1.1.2, ...), 1.2, etc. (the abstract is not included in section numbering). Use this numbering also for internal cross-referencing: do not just refer to 'the text'. Any subsection may be given a brief heading. Each heading should appear on its own separate line.

1.1.2 Format

All text of the manuscript must be located within a 170 mm by 252 mm rectangle of a white A4 page or within 170 mm by 240 mm for the letter format. The margins are given in Table 1. An example of the page format is given in Fig. 1

[Table 1]: ***Page margin for manuscripts.***

Margin Position	Top	Bottom	Left	Right
Margin size (cm)	2.0	2.5	2.0	2.0

All text should be single spaced, black and in 12-point type. "Times News Roman" or a similar proportional font should be used. Total length 15 pages in Word.

The terminology given in the *IEC Technical Report for the Nomenclature of Hydraulic Machinery* is recommended.

Introduction

State the objectives of the work and provide an adequate background, avoiding a detailed literature survey or a summary of the results.

Material and methods

Provide sufficient details to allow the work to be reproduced. Methods already published should be indicated by a reference: only relevant modifications should be described.

Theory/calculation

A Theory section should extend, not repeat, the background to the article already dealt with in the Introduction and lay the foundation for further work. In contrast, a Calculation section represents a practical development from a theoretical basis.

Results

Results should be clear and concise.

Discussion

This should explore the significance of the results of the work, not repeat them. A combined Results and Discussion section is often appropriate. Avoid extensive citations and discussion of published literature.

Conclusions

The main conclusions of the study may be presented in a short Conclusions section, which may stand alone or form a subsection of a Discussion or Results and Discussion section.

References

Within the text, references should be cited in numerical order according to their order of appearance. The numbered reference citation within text should be enclosed in brackets.

After the second edition all papers must have at least one reference of the American Journal of Hydropower, Water and Environment Systems.

Example: It was shown by Prusa [1] that the width of the plume decreases under these conditions.

In the case of two citations, the numbers should be separated by a comma [1,2]. In the case of more than two references, the numbers should be separated by a dash [5-7].

List of References. References to original sources for cited material should be listed together at the end of the paper; footnotes should not be used for this purpose. References should be arranged in numerical order according to the sequence of citations within the text. Each reference should include the last name of each author followed by his initials.

(1) Reference to journal articles and papers in serial publications should include:

- last name of each author followed by their initials
- year of publication
- abbreviated title of publication in which it appears
- full title of the cited article in quotes, title capitalization
- volume number (if any) (Do not include the abbreviation, "Vol.")
- issue number (if any) in parentheses (Do not include the abbreviation, "No.")
- inclusive page numbers of the cited article (include "pp.")

(2) Reference to textbooks and monographs should include:

- last name of each author followed by their initials
- year of publication
- titles in examples may be in italic
- publisher
- city of publication
- inclusive page numbers of the work being cited (include "pp.")
- chapter number (if any) at the end of the citation following the abbreviation, "Chap."

(3) Reference to individual conference papers, papers in compiled conference proceedings, or any other collection of works by numerous authors should include:

- last name of each author followed by their initials
- year of publication
- full title of the cited paper in quotes, title capitalization
- individual paper number (if any)
- full title of the publication
- initials followed by last name of editors (if any), followed by the abbreviation, "eds."
- publisher
- city of publication
- volume number (if any) in boldface if a single number, include, "Vol." if part of larger identifier (e.g., "PVP-Vol. 254")
- inclusive page numbers of the work being cited (include "pp.")

(4) Reference to theses and technical reports should include:

- last name of each author followed by their initials
- year of publication
- full title in quotes, title capitalization
- report number (if any)
- publisher or institution name, city

Sample References

[1] Ning, X., and Lovell, M. R., 2002, "On the Sliding Friction Characteristics of Unidirectional Continuous FRP Composites," ASME J. Tribol., 124(1), pp. 5-13.

[2] Barnes, M., 2001, "Stresses in Solenoids," J. Appl. Phys., 48(5), pp. 2000–2008.

[3] Jones, J., 2000, Contact Mechanics, Cambridge University Press, Cambridge, UK, Chap. 6.

[4] Lee, Y., Korpela, S. A., and Horne, R. N., 1982, "Structure of Multi-Cellular Natural Convection in a Tall Vertical Annulus," Proc. 7th International Heat Transfer Conference, U. Grigul *et al.*, eds., Hemisphere, Washington, DC, 2, pp. 221–226.

[5] Hashish, M., 2000, "600 MPa Waterjet Technology Development," High Pressure Technology, PVP-Vol. 406, pp. 135-140.

[6] Watson, D. W., 1997, "Thermodynamic Analysis," ASME Paper No. 97-GT-288.

[7] Tung, C. Y., 1982, "Evaporative Heat Transfer in the Contact Line of a Mixture," Ph.D. thesis, Rensselaer Polytechnic Institute, Troy, NY.

[8] Kwon, O. K., and Pletcher, R. H., 1981, "Prediction of the Incompressible Flow Over A Rearward-Facing Step," Technical Report No. HTL-26, CFD-4, Iowa State Univ., Ames, IA.

[9] Smith, R., 2002, "Conformal Lubricated Contact of Cylindrical Surfaces Involved in a Non-Steady Motion," Ph.D. thesis, http://www.cas.phys.unm.edu/rsmith/homepage.html

1.1.2 Essential title page information

- **Title.** Concise and informative. Titles are often used in information-retrieval systems. Avoid abbreviations and formulae where possible.
- **Author names and affiliations.** Where the family name may be ambiguous (e.g., a double name), please indicate this clearly. Indicate all affiliations with a number immediately after the author's name and in front of the appropriate address. Provide the full postal address of each affiliation, including the country name and, if available, the e-mail address of each author.
- **Author résumé.** The author must inform the graduation degree, post graduation, affiliation and email address. The résumé must not exceed 150 characters.
- **Corresponding author.** Clearly indicate who will handle correspondence at all stages of refereeing and publication, also post-publication. Ensure that e-mail address and the complete postal address are provided. Contact details must be kept up to date by the corresponding author.
- **Present/permanent address.** If an author has moved since the work described in the article was done, or was visiting at the time, a 'Present address' (or 'Permanent address') may be indicated as a footnote to that author's name. The address at which the author actually did the work must be retained as the main, affiliation address. Superscript Arabic numerals are used for such footnotes.

Abstract

A concise and factual abstract is required. The abstract should state briefly the purpose of the research, the principal results and major conclusions. An abstract is often presented separately from the article, so it must be able to stand alone. For this reason, References should be avoided, but if essential, then cite the author(s) and year(s). Also, non-standard or uncommon abbreviations should be avoided, but if essential they must be defined at their first mention in the abstract itself.

Keywords

Immediately after the abstract, provide a maximum of 6 keywords, using American spelling and avoiding general and plural terms and multiple concepts (avoid, for example, 'and', 'of'). Be sparing with abbreviations: only abbreviations firmly established in the field may be eligible. These keywords will be used for indexing purposes.

Abbreviations

Define abbreviations that are not standard in this field in a footnote to be placed on the first page of the article. Such abbreviations that are unavoidable in the abstract must be defined at their first mention there, as well as in the footnote. Ensure consistency of abbreviations throughout the article.

Acknowledgements

Collate acknowledgements in a separate section at the end of the article before the references and do not, therefore, include them on the title page, as a footnote to the title or otherwise. List here those individuals who provided help during the research (e.g., providing language help, writing assistance or proof reading the article, etc.).

Nomenclature and units

Follow internationally accepted rules and conventions: use the international system of units (SI). If other quantities are mentioned, give their equivalent in SI.

Math formulae

Present simple formulae in the line of normal text where possible and use the solidus (/) instead of a horizontal line for small fractional terms, e.g., X/Y. In principle, variables are to be presented in italics. Powers of e are often more conveniently denoted by exp. Number consecutively any equations that have to be displayed separately from the text (if referred to explicitly in the text).

Footnotes

Footnotes should be used sparingly. Number them consecutively throughout the article, using superscript Arabic numbers. Many wordprocessors build footnotes into the text, and this feature may be used. Should this not be the case, indicate the position of footnotes in the text and present the footnotes themselves separately at the end of the article. Do not include footnotes in the Reference list.

Table footnotes

Indicate each footnote in a table with a superscript lowercase letter.

Artwork

Electronic artwork

General points

- Make sure you use uniform lettering and sizing of your original artwork.
- Save text in illustrations as 'graphics' or enclose the font.
- Only use the following fonts in your illustrations: Arial, Courier, Times, Symbol.
- Number the illustrations according to their sequence in the text.
- Use a logical naming convention for your artwork files.
- Provide captions to illustrations separately.
- Produce images near to the desired size of the printed version.
- Submit each figure as a separate file.
- Pictures, graphics and images must be submitted in a JPG or GIF format with 300 dpi.

2 Conducting the Review

2.1 Originality

You might wish to do a quick literature search using tools such as Scopus to see if there are any reviews of the area. If the research has been covered previously, pass on references of those works to the editor.

2.2 Structure

Consider each element in turn: Title; Abstract; Introduction (It should describe the experiment, the hypothesis(es) and the general experimental design or method); Method; Results; Conclusion/Discussion; Language: you do not need to correct the English. You should bring this to the attention of the editor, however.

2.3 Previous Research

If the article builds upon previous research does it reference that work appropriately? Are there any important works that have been omitted? Are the references accurate?

2.4 Ethical Issues

Plagiarism: If you suspect that an article is a substantial copy of another work, please let the editor know, citing the previous work in as much detail as possible

Fraud: It is very difficult to detect the determined fraudster, but if you suspect the results in an article to be untrue, discuss it with the editor

AUTHORIZATION FOR PUBLICATION OF PAPERS

LICENSE FOR USE OF INTELLECTUAL WORK (Author)

For this private instrument the AUTHOR, below signed authorizes the IAHR Latin American Working Group, to publish its work authorship, without any obligation and in exclusiveness character for the period of six months starting from the publication in the AMERICAN JOURNAL OF HYDROPOWER, WATER AND ENVIRONMENT SYSTEMS, or in another official publication of IAHR.

In case of joint authorship, the first author signs as AUTHOR, assuming before IAHR the commitment of informing the other authors of the granted license.

AUTHOR (full name in form letter):

Title of the Paper:____

JOINT AUTHORS [full name in form letter]:______________________________

ADDRESS:____

____.

Email:____

IAHR AIRH

IAHR DIVISION I: HYDRAULICS
TECHNICAL COMMITTEE: HYDRAULIC MACHINERY AND SYSTEMS

American Journal of Hydropower, Water and Environment Systems

Number 5 **Out 2017**

Technical Papers

American Journal of Hydropower, Water and Environment Systems
Publisher: Acta Editora/LAWG-IAHR
DOI:10.14268/ajhwes.2017.00047
ISSN: 2317-126X
Subject Collection: Engineering,
Subject: engineering, measurement, environment Systems

Hydraulic Transients in Penstocks: Comparison of Methods Runge-Kutta and Characteristics in Load Rejection Solution

[1]Marra, João M.; [2]Gramani, Liliana M.; [3]Santos, Christian W.; [4]Kaviski, Eloy.

ABSTRACT

This paper presents a comparative study of results obtained by the methods of Characteristics and Runge-Kutta in the numerical solution of governing equations for determining the pressure behavior in the hydraulic system of a Francis turbine during transients in its flow. For this purpose is analyzed the effects of using different discretizations for space and time in the results in both these methods. Also, a formulation that avoids numerical instability in the Method of Characteristics in hydraulic systems modeled with variable geometry is tested. In the solution for Runge-Kutta, the representation of hydraulic systems is made through equivalent electric circuits. The results validation is based on available data of transients recorded during load rejection testing in the Francis turbines of Itaipu Power Plant and has indicated that for a suitable mesh space-time the Runge-Kutta method presents accuracy and speed of processing that configures it as an alternative to the traditional Method of Characteristics for this type of estimation. Additionally, the equating used at the Method of Characteristics allowed to apply it for a pipeline with variable diameter without numerical instability was observed. As an application of this study is analyzed the possibility of changing the distributor's closing time law of these turbines, that provides more favorable values of overpressure due to water hammer and overspeed in the generating unit in load rejections using a more realistic representation of the hydraulic system of the turbine.

KEYWORDS: hydraulic transient, numerical simulation, water hammer, penstock, turbine.

1. INTRODUCTION

The regularization of an interconnected electrical system is a complex process and requires instant and permanent action to equilibrate the natural oscillations and abrupt variations the load with generation. Also, should be equilibrated the swings and sudden changes provoked by equipment failure or temporary lack of energetic availability of some source as, for example, the wind and solar power, which increasingly are present in the Brazilian and world energy matrix.

In this context, the hydroelectric power plants are versatile in meeting the load variations and of the interconnected power system generation, due to the rapidity of power's response due to a favorable ratio of rotational inertia and hydraulic response. However, in meeting these variations, often these plants operate outside their optimal hydraulic conditions, including due to seasonal variation in hydraulicity or hydraulic crises, increasingly frequent by global warming. In this scenario, the machines of simple regulation how the Francis turbines, responsible for significant contribution in the hydroelectric generation in Brazil and the world, and also for 60% of the world hydraulic potential to be installed, are usually more sensible due its intrinsic characteristics, mainly concerning the efficiency and disturbances in the flow.

The knowledge of the pressure behavior in the hydraulic system of a hydraulic turbine during flow transients is fundamental in the penstock and generating unit design stage, and the correct estimate of this represents challenges due to the complexity of the actual installation of a hydroelectric plant. The pressure variations caused by water hammer in an abrupt load change could be quantified with accuracy through the Method of Characteristics. However, for oscillations associated the phenomena of resonance or hydraulic instability during normal operation of the turbine, the representation of hydraulic systems through equivalent electric circuits solved by the numerical integration of Runge-Kutta presents some advantages in the mathematical modeling of the hydraulic system.

To verify the accuracy and speed time processing of the methodology indicated in this paper is made a comparative study of the numerical results using different space and time mesh discretizations for both methods, and also existing measuring data from a load rejection of an original hydroelectric plant.

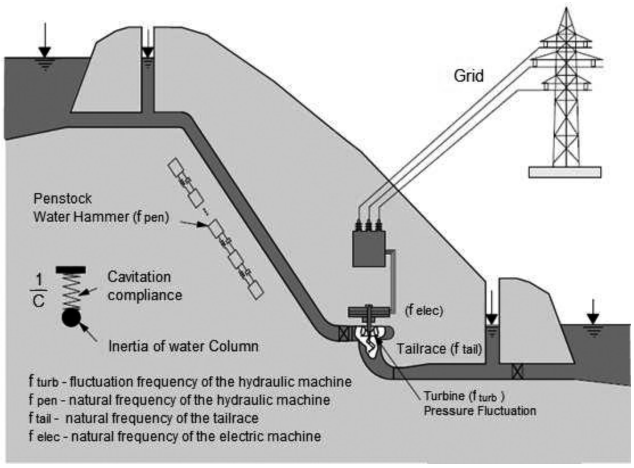
Beyond the numerical verification of the guaranteed values of overpressure and overspeed during the design stage, another applic designer to make an optimization of the closing time law, reducing the experimental runs for reach this purpose.

2. GOVERNING EQUATIONS AND ELECTRICAL ANALOGY

The hydraulic circuit of hydroelectric pl plants is characterized by having much larger longitudinal dimension than the transversal, as an illustration of Fig. 1. In function of this typical configuration, the working fluid flow has predominant characteristics in the longitudinal direction and negligible temperature variation, allowing a representative one dimensional mathematical modeling of the dynamic flow behavior based on the momentum and mass conservation laws.

According to [2], the application of Newton's second law on a free body diagram of forces to an elastic element of dx length of the pipeline and continuity equation to the same element subjected to a hydraulic piezometric line results on the following governing equations system for the transient one-dimensional flow on the pipeline element, where 'a'

[1]Department of Maintenance Engineering, Itaipu Binational, Iguassu Falls, 85856-970, Brazil, e-mail: marra@itaipu.gov.br; Tel: (+55 45)3520-2690; Fax: (+55 45)3520-3835
[2]Department of Mathematics, Federal University of Parana - UFPR, Curitiba, 81531-990, Brazil, e-mail: gramani@ufpr.br
[3]Center for Advanced Studies in Dam Safety – FPTI, Iguassu Falls, 85.867-900, Brazil, e-mail: christian.santos@pti.org.br
[4]Department of Hydraulic and Hydrology, Federal University of Parana – UFPR, Curitiba, 81531-990, Brazil, e-mail: eloy.dhs@ufpr.br



[Figure 1: Scheme of typical hydraulic system of a hydroelectric plant ([1])]

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{fQ|Q|}{2gDA^2}=0 \tag{1}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{2}$$

On the above system the Eq. (1) is referred to the momentum equation and the Eq. (2) to a transport equation given by the mass conservation. The equations system formed is a hyperbolic first order non-linear partial differential equations system, for which the method of characteristics, based on finite differences, is traditionally used in its numerical solution for given initial and boundary conditions.

As stated at [3], the equations system above is analogous to electric wave propagation in electric conductors, wherein the flow *Q* corresponds to the electric current, and piezometric head *H* (or pressure) corresponds to voltage. Based on the analogy of the two systems, the parameters correspondents of the hydraulic system are qualified to traditional denominations of parameters *R* (Resistance), *L* (Inductance) and *C* (Capacitance) for an electric system, as indicated below:

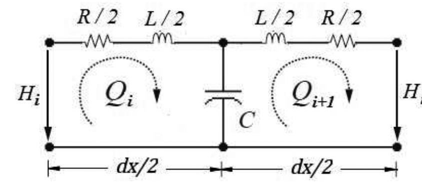
$$\frac{\partial H}{\partial x}+L'\frac{\partial Q}{\partial t}+R'(Q)Q=0 \tag{3}$$

$$\frac{\partial H}{\partial t}+\frac{1}{C'}\frac{\partial Q}{\partial x}=0 \tag{4}$$

In the hydraulic system the parameter *R* represents the loss of energy by dissipative effects, *L* and *C* represents, respectively, the effects of inertia and storage in volume. The parameter *C* is also denominated compliance because the storage effect is due to the fluid compressibility and the pipeline elasticity. Due to the dependence of the resistance *R* with the flow rate *R(Q)*, the partial differential equations system (Eq. 3 and Eq. 4) is nonlinear. The apostrophe signal indicates that the values of parameters in the equations are per length unit:

$$R'=\frac{f|Q|}{2gDA^2}\left[s/m\right] \qquad L'=\frac{1}{gA}\left[s^2/m^3\right] \qquad C'=\frac{gA}{a^2}\left[m^2\right] \tag{5}$$

The circuit of the hydraulic system equivalent to the electric circuit RLC is indicated on Fig. 2, where the index *i* and *i+1* represents the state variables value *(H, Q)* at the opposite ends of the element considered.



[Figure 2: Scheme of the equivalent electric circuit of the elastic tube element Adapted from [4]).]

In function of the analogy between the hydraulic and electrical circuits, the governing equations of the hydraulic circuit can be obtained by applying the laws of Kirchoff and the law relating to the electrical voltage drop (or hydraulic pressure) on the elements of the circuit, as indicated on the Tab. 1, where *I* é the current and *U* the voltage.

[Table 1]: ***Analogy of electrical and hydraulic circuits.***

Law	Application	Electric	Hydraulic
1st Kirchhoff's law	Node law	$\sum_{i=1}^{n} I_i = 0$	$\sum_{i=1}^{n} Q_i = 0$
2nd Kirchhoff's law	Mesh law	$\sum_{i=1}^{n} U_i = 0$	$\sum_{i=1}^{n} H_i = 0$
Ohm's law	Voltage drop on resistor	$\Delta U = RI$	$\Delta H = RQ$
Lenz's law	Voltage drop on inductor	$\Delta U = L\frac{dI}{dt}$	$\Delta H = L\frac{dQ}{dt}$
Capacitance	Voltage drop on capacitor	$I = C\frac{dU}{dt}$	$Q = C\frac{dH}{dt}$

3. SPATIAL DISCRETIZATION OF HYDRAULIC SYSTEM AND MODELING OF PIPELINE

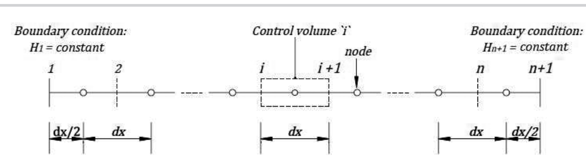
In this simulation, all of the hydraulic pipeline stretches are modeled as elastics elements of steel, regardless of these being installed in apparent steel, embedded or concrete only.

3.1 For the Runge-Kutta method – RKM

For a generic pipe of length *l*, applying a discretization based on a central scheme, it can quantify the spatial variation of manometric height *H*, flow rate *Q* and the mean flow value at the node i+1/2, as, respectively, in the expressions indicated on Eq. (6):

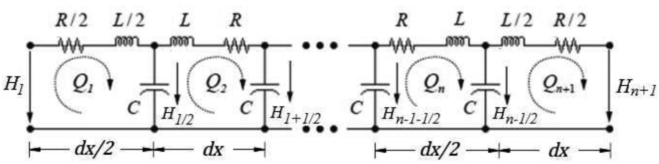
$$\left.\frac{\partial H}{\partial x}\right|_{i+1/2}=\frac{H_{i+1}-H_i}{dx} \quad ; \quad \left.\frac{\partial Q}{\partial x}\right|_{i+1/2}=\frac{Q_{i+1}-Q_i}{dx} \quad ; \quad Q_{i+1/2}=\frac{Q_{i+1}+Q_i}{2} \tag{6}$$

The central scheme used at spatial discretization for the pipeline length l is shown at Fig. 3:



[Figure 3: One-dimensional spatial discretization of pipeline (Adapted from [4]).]

For the considered elastic tube, the scheme of the equivalent electric circuit to the adopted discretization of the hydraulic system is shown in Fig. 4:



[Figure 4: Elastic tube equivalent electric circuit (Adapted from [4]).]

Substituting the expressions of Eq. 6 in Eq. (3) and Eq. (4) and making *R' dx = R, L' dx = L e C' dx = C is* obtained an ODE system for one element of an elastic tube of length dx, that expressed in matrix form becomes:

$$\begin{pmatrix} C & 0 & 0 \\ 0 & L/2 & 0 \\ 0 & 0 & L/2 \end{pmatrix} \cdot \frac{d}{dt} \begin{pmatrix} H_{i+1/2} \\ Q_i \\ Q_{i+1} \end{pmatrix} + \begin{pmatrix} 0 & -1 & 1 \\ 1 & R/2 & 0 \\ -1 & 0 & R/2 \end{pmatrix} \begin{pmatrix} H_{i+1/2} \\ Q_i \\ Q_{i+1} \end{pmatrix} = \begin{pmatrix} 0 \\ H_i \\ -H_{i+1} \end{pmatrix} \quad (7)$$

In compact matrix form, the equation system (7) is reduced to Eq. (8), where $\vec{x} = (H_{i+1/2},\ Q_i,\ Q_{i+1})$ is the state variable vector of the discretized system and A and B are 3x3 matrices:

$$[A] \cdot \frac{d\vec{x}}{dt} + [B] \cdot \vec{x} = \vec{C} \quad (8)$$

For the elastic tube of length l discretized as indicated in Fig 3, the matrices [A] and [B] of (Eq. 11) have order *2n + 1* and the vectors of state $\vec{x}$ (Eq. 9) and of boundary conditions $\vec{C}$ (Eq. 10) have dimension (2n +1, 1):

$$\vec{x} = \left(H_{1+1/2} \quad H_{2+1/2} \quad \cdots \quad H_{n+1/2} \quad Q_1 \quad Q_1 \quad \cdots \quad Q_{n+1} \right)^T \quad (9)$$

$$\vec{C} = \left(0 \quad 0 \quad \cdots \quad H_1 \quad 0 \quad 0 \quad \cdots \quad H_{n+1} \right)^T \quad (10)$$

$$[A] = \begin{pmatrix} C & & & & & & & (2n+1) \\ & C & & & & & & \\ & & \ddots & & & 0 & & \\ & & & C & & & & \\ (n) & & & & C & & & \\ & & & & & L/2 & & \\ & & & & & & L & \\ & & 0 & & & & & \ddots \\ & & & & & & & L \\ (2n+1) & & & & & & & L/2 \end{pmatrix} \quad [B] = \begin{pmatrix} & & & & -1 & 1 & & (2n+1) \\ & & & & & -1 & 1 & \\ & 0 & & & & & \ddots & \\ & & & & & & -1 & 1 \\ (n) & & & & & & & -1 \quad 1 \\ 1 & & & R/2 & & & & \\ -1 & 1 & & & R & & & \\ & \ddots & & & & \ddots & & \\ & & -1 & 1 & & & R & \\ (2n+1) & & & -1 & & & & R/2 \end{pmatrix} \quad (11)$$

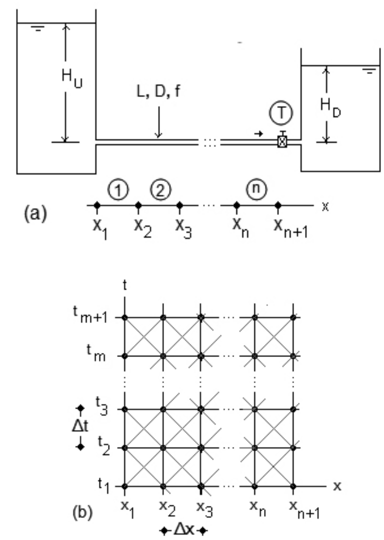
At this work, the Runge-Kutta fourth order method in explicit form will be used for the integration of the Eq. (8), whose solution makes possible to know the time evolution of the pressure and flow rate at each node of interest of the mesh of the system during a hydraulic transient.

3.2 For the method of characteristics – MOC

The method of characteristics can be used widely to solve initial value problem relative to first order ODE and that when applied to a PDE with two variables the change of coordinates of the process transforms the PDE in ODE along certain curves called Characteristic Curves, over which the new variables will be constant in these curves, as stated in [5]. The application of the method of characteristics on the governing equations (1) and (2) results in two ODE systems (Eq. 12) correspondents to two characteristics equations relatives to the propagation of the wave flow positive direction (C+) and on opposite direction (C-). Conforming to [3], such systems of equations are:

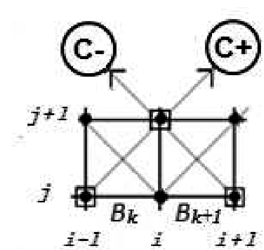
$$\text{C+:} \begin{cases} \dfrac{dH}{dt} + \dfrac{a}{g.A}\dfrac{dQ}{dt} + \dfrac{a}{g}\dfrac{f.Q.|Q|}{2.D.A^2} = 0 \\ \dfrac{dx}{dt} = +a \end{cases} \qquad \text{C}-\text{:} \begin{cases} \dfrac{dH}{dt} - \dfrac{a}{g.A}\dfrac{dQ}{dt} - \dfrac{a}{g}\dfrac{f.Q.|Q|}{2.D.A^2} = 0 \\ \dfrac{dx}{dt} = -a \end{cases} \quad (12)$$

The spatial discretization of the hydraulic system and the spatial-temporal adopted for the method of characteristics are respectively illustrated in Fig. 5.a and Fig. 5.b. On this process, waves travel with the celerity a along the characteristic lines, represented on the plane (x, t) by diagonals lines, obeying to the relationship a = $\Delta x/\Delta t$.



[Figure 5: a) Spatial discretization of hydraulic [6]; b) Spatial and temporal discretization [6].]

According to [3] and [7], applying progressive finite differences to the derivatives of equations (12), integrating over the positive characteristic lines (C+) and negatives ones (C-), illustrated in detail in Fig. 6, and adapting their solution for a penstock with variable diameter is obtained the expressions of Eq. (13), where $B_k = (a/g.A_k)$ and $R_k = f/(2.g.D.A_k^2)$ and the generalized position of the nodes at the space-time plane is indicate by the indexes *i* and *j*.



[Figure 6: Characteristics curves (Adapted from [6]).]

$$\begin{cases} H_i^{j+1} + B_k.Q_i^{j+1} = H_{i-1}^j + B_k.Q_{i-1}^j - R_k.Q_{i-1}^j.\left|Q_{i-1}^j\right|(\Delta x) = C_P \\ H_i^{j+1} - B_{k+1}.Q_i^{j+1} = H_{i+1}^j - B_{k+1}.Q_{i+1}^j + R_{k+1}.Q_{i+1}^j.\left|Q_{i+1}^j\right|(\Delta x) = C_N \end{cases} \quad (13)$$

By solving the linear system represented by Eq. (13), is obtained the expression of Eq. (14) that quantifies respectively the amplitudes of pressure and flow rate in the hydraulic system in the discretized domain:

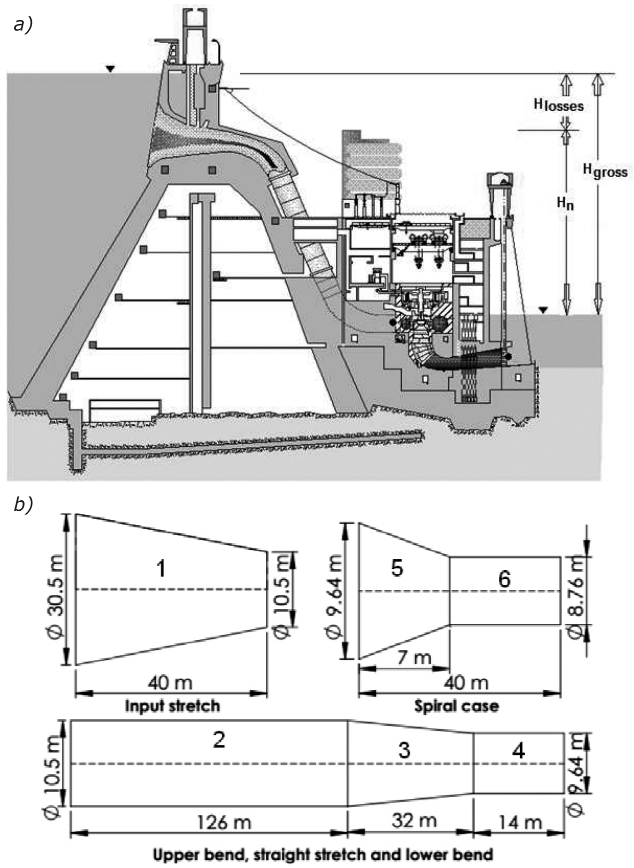
$$H_i^{(j+1)} = (B_{k+1}C_P + B_k C_N)/(B_{k+1} + B_k) \qquad Q_i^{(j+1)} = (C_P + C_N)/(B_k + B_{k+1}) \quad (14)$$

The use of Eq. (14) is an attempt to apply the MOC in modeling with geometric variation (diameter) of the pipeline of the hydraulic system, since the use of traditional expressions shown in Eq. (15) leads to numerical instability when the system is not modeled with constant geometry as indicated in Fig. 5.a or to the necessity of using of a very refined spatial discretization to control the inherent instability, which increases severely the processing time.

$$H_i^{(j+1)} = (C_P + C_N)/2 \qquad Q_i^{(j+1)} = (C_P + C_N)/(2.B) \quad (15)$$

4. MODELING OF HYDRAULIC SYSTEM

This section discusses the geometric modeling the variation of the diameter of the pipe along the hydraulic system of the turbines of ITAIPU Power Plant shown at Fig. 7.a, from the water to the outlet end of the spiral casing and also the modeling of the functional elements which were considered as concentrates (lumped), as shaft surge tank and turbine distributor.



[Figure 1: a) Hydraulic system of ITAIPU Power Plant [8]; b) Geometric modeling]

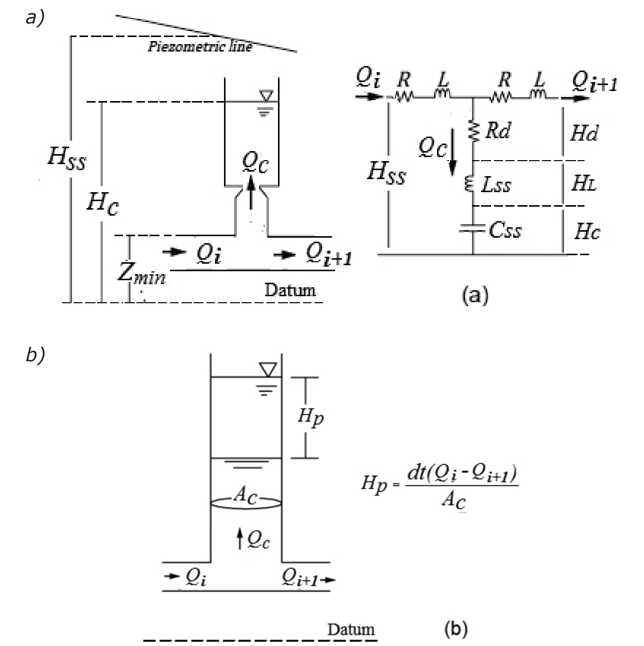
4.1 Pipeline

The variation of the penstock diameter at the input stream, the upper curve, straight stretch, lower curve and spiral casing hydraulic turbines of ITAIPU is shown in Fig. 7.b.

The coefficient of friction in hydraulic surfaces was obtained from data available of pressure drop in the hydraulic system and of the application of Darcy-Weisbach relationship, resulting in an average value of 0.025

4.2 Shaft surge tank

The duct to the purging of atmospheric air in the turbine and penstock priming and the niches of the entrance gate and of the stop-log existing on input stret normal operation or load rejection. The adopted mathematical model for this element on RKM and MOC are shown in Fig. 8.a and Fig. 8.b, respectively.



[Figure 1: Mathematical model of the shaft surge tank: a) RKM(Adapted from [4]); b) MOC (Adapted from [2])]

Solving the circuit obtains the Eq. (16) compatible with the discretization and state vector adopted in the application of Runge-Kutta for this element of the hydraulic system:

$$\begin{bmatrix} C_{ss} & 0 & 0 \\ 0 & (L+L_{ss}) & -L_{ss} \\ 0 & -L_{ss} & (L+L_{ss}) \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} H_c \\ Q_i \\ Q_{i+1} \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ -1 & (R+R_d) & -R_d \\ 1 & -R_d & ((R+R_d)) \end{bmatrix} \cdot \begin{bmatrix} H_c \\ Q_i \\ Q_{i+1} \end{bmatrix} = \begin{bmatrix} 0 \\ H_{ss} \\ -h_{i+1} \end{bmatrix} \quad (16)$$

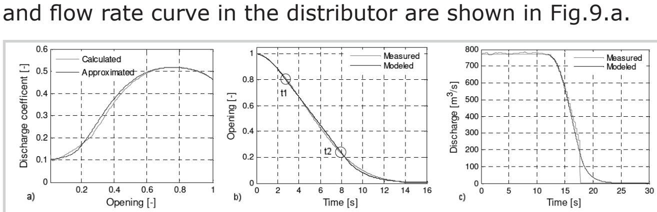
In the case of the method of characteristics, the modeling used for the ventilation duct was only based on the continuity equation, resulting in the equation indicated in the illustration of Fig.8.b.

4.3 Wicket gate

The discharge coefficient curve C_D in function of the free area A_W of the turbine distributor was calculated using the relation of Eq (17). Distributor area A_W, flow rate Q_W, and the pressure H_n were obtained from an existing recorded digital data of simultaneous measurements at a full gate load rejection test performed at the commissioning of the turbine U18A in 2005. Calculated and approximated curves of C are indicated in Fig. 9.a. The flow rate measurement was based on Winter-Kennedy method, at whose constant was later determined using two absolute methods, Gibson and Ultrasonic by transit time. Due to the existence of a cut-off on the flow transducer near 250 m³/s, the flow rate curve was completed respecting the closing time law indicated in Fig. 9.b, resulting in the modeled flow rate curve used of the Fig. 9.c.

$$C_D = Q_W \Big/ \left(A_W \sqrt{2g.H_n} \right) \quad (17)$$

The curves for the discharge coefficient, closing time law and flow rate curve in the distributor are shown in Fig.9.a.

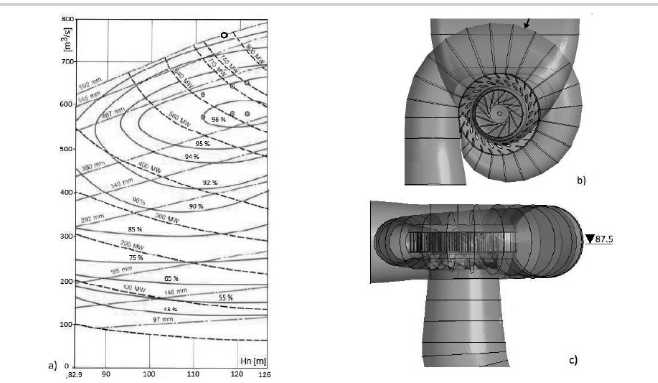


[Figure 9: a) Discharge coefficient; b) Closing time law; c) Flow curve.]

During simulation of the hydraulic transient, the flow rate Q in the distributor was obtained using the Eq. (18), where b is the height of the vanes, Z is the number of vanes and S is the opening related to the distributor.

$$Q_t = C_D . A_t \sqrt{2g.H_n} = C_D .\left(Z \cdot b \cdot S\right)\sqrt{2g.H_n} \tag{18}$$

According to [9], it's necessary to perform series of computation to evaluate the influence of the turbine's distributor closing time on guaranteed control values. To facilitate the analysis of the influence of the variation of closing time law in overpressure and overspeed, this was modeled considering the possibility of changing the closing time by changing the opening at t_1 and the opening and time at t_2 for any given distributor initial opening in t_0, as illustrated in Fig. 9.b. The initial (0-t1) stretch of the curve of Fig.9.b. was modeled by a polynomial, the intermediate stretch between t_1 and t_2 by a straight line and the final stretch by an exponential curve. The flow rate curve of Fig. 9.c refers to the studied case with an initial discharge of 770 m^3/s, very close the maximum possible according to the hill chart of the Fig. 10.a about the ITAIPU turbines, considering the 800 MW power limitation existing at these turbines. The measured pressure considered to comparison with simulated values was taken on a piezometric tap positioned at spiral case door at its mean elevation, as indicated in Fig. 10.b and Fig. 10.c.



[Figure 9: a) Hill Chart of the Itaipu turbines; b) Pressure measuring point; c) Elevation of the pressure tap.]

At the application of the present study was considered an installed power plant with data of measurement available to make possible an evaluation of the methodology used. However, a similar procedure may be used independently if the plant is already installed or still under design. Therefore, the discharge variation may be obtained numerically introducing at the equivalent electric circuit a downstream tip element representing the distributor how a lumped element with hydraulic resistance R_d and head losses H_d , as described at [4], and the vane opening S obtained by the preview distributor closing time law established.

4.4 Rotating parts

The sudden load rejection of a generating unit for decoupling thereof with the electrical system provides an unbalancing of torque between the turbine and generator, and a consequent increase in speed. Because of this, the turbine speed governor commands the closing of the turbine distributor, which in turn causes a transient hydraulic pressure in the penstock, known as water hammer. In this condition, assuming that the magnetic torque in the air gap torque in the generator cancels instantly, the rotation speed N of the rotating assembly becomes dominated by the mechanical torque T_{mec} and the rotational inertia J, according to Eq. (19):

$$T_{mec} - T_{mag} = \left(\frac{2\pi}{60}\right) J \frac{dN}{dt} \tag{19}$$

Equation 18 can be integrated by separation of variables, whose discrete approximation of the solution given by Eq. (20) allows to obtaining the evolution of the rotation during the transient from the rejected power the turbine at rated speed. During the transient, turbine shaft power was evaluated by the expression $P = \gamma Q_t H_n \eta$, but without consider changing at the efficiency with the rotation variation. The turbine efficiency was obtained of its hill chart. The moment of inertia was achieved by existing measurements of the GD^2 factor realized at type acceptance tests of the ITAIPU 60 Hz hydrogenerators, whose value was considered 328150 tm^2, equivalent to 8.380E6 kg m^2.

$$N_{t+1} = \sqrt{N_t^2 + \left(\frac{60}{2\pi}\right)^2 \frac{\Delta t}{J}\left(\left(P_{t+1} + P_t\right) - \left(P_{b(t+1)} + P_{b(t)}\right)\right)} \tag{20}$$

The intrinsic braking power P_b was obtained by assuming a linear variation of the losses dependent on voltage generator (2.1 MW) with rotation and a cubic variation with the rotation for the friction losses (2.1 MW) in the bearings and ventilation.

5. MESH SPACE AND TIME AND COMPUTING RESOURCES

The numerical simulations by the MOC require that minimum time interval in computational iterations respects the criterion of Courant-Friedrich-Levy, which establishes that the Courant number $C_r = a \cdot \Delta t/\Delta x = a \cdot n \cdot \Delta t/l$ must be equal to the unit for an explicit method. Although the RKM method in the explicit application doesn't be unconditionally stable, this requirement isn't necessary. For RKM the minimum Courant was 0.05 and the maximum was 1.0.

To evaluate the accuracy of both methods in representing the transient phenomenon and the impact of discretizations adopted in processing time, this study was conducted considering 8 different spatial discretizations (M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8) and 4 steps of time (T_1, T_2, T_3 and T_4), totaling 32 different configurations for mesh spatial and temporal. From a total of spatial discretizations, 4 were with uniform length elements and 4 with a non-uniform length of elements to reduce the total number of elements, for which was adopted a different discretization, but with homogeneous elements for the straight sections (2, 4, 6) of the modeled pipeline in Figure 7.b. The steps of time adopted aimed to work with unitary Courant at the MOC, considering the smaller spatial mesh element and maintained a constant ratio of 0.5 (T1/T2), 0.4 (T2/T3) and 0.25 (T3/T4) between the respective steps of each spatial configuration.

Thus, for example, the mesh M1.T1 has spatial discretization M1 and step time T1 corresponding to this discretization and so on for the others combinations of time and space. The spatial configuration adopted is shown in Table 2, where the length of the elements per stretch is in meters.

[Table 2]: Spatial discretization.

Stretch	M1	M2	M3	M4	M5	M6	M7	M8
1	0,5	1	2	3	1	1	2	4
2	0,5	1	2	3	10,5	63	63	63
3	0,5	1	2	3	1	1	2	4
4	0,5	1	2	3	7	7	7	7
5	0,5	1	2	3	1	1	2	4
6	0,5	1	2	3	9,5	18	18	18
Total of Elements	510	255	128	85	95	83	45	26

All simulations were performed on a notebook type computer with a dual-core processor, CPU 1.6 GHz / 2.6 GHz and 4GB of RAM. The variation of the processing time was 5:1 with respect to duplication of the element length and 1:2 doubling the time step. Therefore, reducing the length of the element by half and doubling the time step resulted in an approximate 10 times reduction in processing time.

6. RESULTS

The present comparative study performed was based on a existing measurements of a sudden load rejection of 780,5 MW, with full opening of the distributor and gross head of 120.2 mWc realized at commissioning tests of the U18A.

Considering the losses in the generator, the rejected power in the turbine shaft was estimated in 791,5 MW, corresponding to flow of 770 m3/s and net head of 118.4 mWc in accordance with the hill curve of the turbine. The maximum pressure P and speed rotation N measured for this condition were 166.4 mWc and 141.8 rpm, corresponding an elevation of 29,4% and 41,7% for these parameters, respectively.

Although 32 different space-time meshes have been simulated, the main results obtained are enough represented at the six cases indicated in Table 3 with relation to the time simulation, percentage relative error to the maximum measured value of the overpressure ΔP and overspeed ΔN and pressure perturbations due to the return of the acoustics waves. Because of this only these six cases are presented in this section. Some parameters of the simulation are also indicated. It was used a constant friction coefficient of 0.025 and a simulated time of the process of 30s for all studied cases. For the case of mesh M6.T1 with non-uniform spatial elements, the Courant value is referred to the smaller element. For the others cases indicated in Table 3, the spatial elements are uniforms.

The time evolution of the pressure obtained by the methods RKM and MOC for the cases indicated in Table 3 are shown in Fig. 11.a to Fig. 11.f, superimposed on a subsampling in 20 Hz of the measured values. The maximum values for overpressure ΔP and overspeed ΔN by both methods are in general equivalents and consistent to the values measured. However, the relative error for the MOC increases sharply for meshes with nonuniform spatial elements and slightly for the RKM with increasing time step, though this is not evident in the cases of Table 3. With respect to processing time, MOC always presents low values for even refined meshes, unlike of RKM that has highest processing time as well as a greater increase of this how much the space-time mesh is finer. However similar results by both methods can be obtained choosing adequately the mesh. For example, similar results to the MOC-M4.T1 (1s) can be obtained at RKM-M1.T4 (77,5h) or RKM-M6.T1 (143s). None of the methods presents numerical instability, signaling that the proposed equating with a variable diameter for the MOC was successful.

Both methods also captured the reduction of the pressure immediately after the start of the transient. This slight reduction in pressure before its rising can be attributed to the inertia of the flow, providing an increase in speed energy at the beginning of the closure of the distributor. With relation the pressure oscillation due overlap of positive and negative waves in the pressure, whose theoretical period is $T = 4L\ a$ = 1.061s or frequency of 1.36 Hz, this was very dependent of the space-time discretization, reaching to be severely masked in RKM for some cases, e. g., RKM-M1.T1.

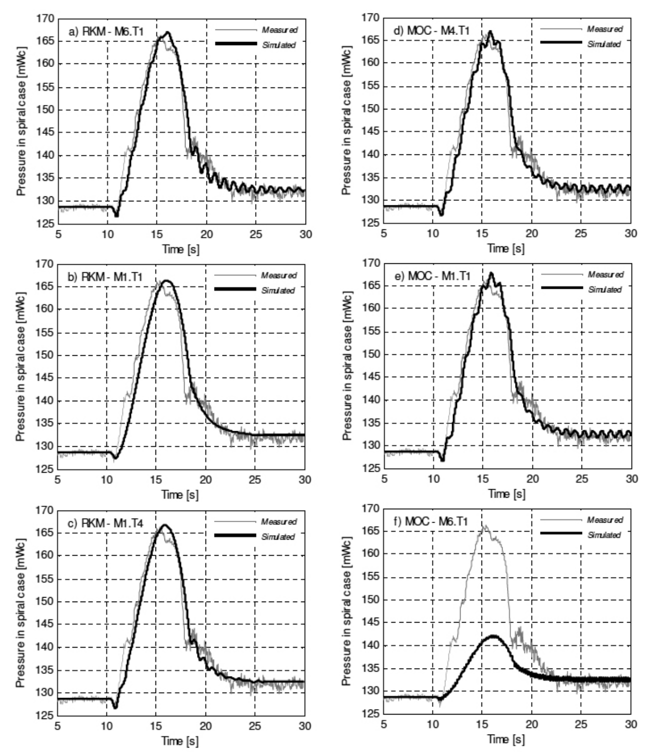
So the coarser non-uniform mesh M6.T1 presented better results than the finer uniform mesh M1.T1. The celerity of the waves was obtained considering the elasticity of the tubing and of the fluid. The actual values found for the period of the pressure oscillations was 0.89s at the numerical simulations and 0.77s at field test. Refining of the modeling of the distributor and consideration of viscoelastic effects could improve the adherence to this parameter.

The simulated overspeed curve of Fig. 12.a may be considered quite equivalent for both methods and cases, except for those where the relative error of the pressure is abnormal, as in case of MOC-M6.T1. A higher value of the simulated rotation and the time lag between measured and simulated values can be attributed mainly to the lack of updating the turbine efficiency with the variation of its rotation, reducing the dissipative braking forces. This deficiency could be corrected introducing at the numerical routine the variation in efficiency with the turbine speed during the hydraulic transient after the load rejection. For this purpose, the use of the characteristic curves of the turbine in its dimensionless representation (polar) shown on [2], [3] and [4] is recommended.

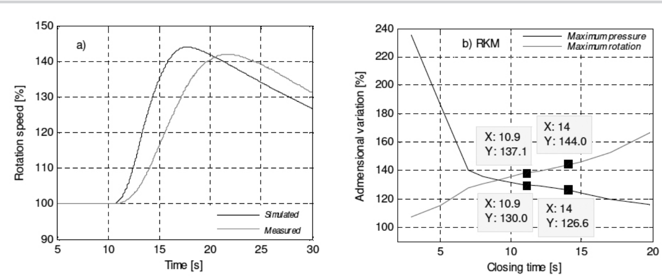
The results of the simulations with variation in the closing time law are shown in Fig. 12.b, in which is evidenced the possibility of reducing approximately 7% in overspeed after a full load rejection, keeping the nominal limit of 30% in overpressure. At the graph of this figure y-axis is non-dimensional variation of the overpressure and overspeed relative the rated (100%) level for the pressure and rotation parameters. Therefore an elevation of 30% corresponds to a level of 130% in the graph curve. So, reducing the current closing time of 14s corresponding the maximum opening (full gate) to about 10.9s would result in an estimated overspeed value of 137.1%, that is around 7.1% below the estimated 144.0% for the current closing time.

[Table 3]: Spatial discretization.

Method/Mesh	l [m]	n	a [m/s]	Δt [s]	Cr	Simulation time	P [mWc]	N [rpm]	ΔP [%]	ΔN [%]
Runge-Kutta – M6.T1	252	83	950	1.05e-3	1	143 [s]	166,9	144,0	0.30	1,55
Runge-Kutta – M1.T1	252	510	950	5.26e-4	1	4.98 [h]	166,4	144,1	0.00	1.62
Runge-Kutta – M1.T4	252	510	950	2.63e-5	0,05	77.5 [h]	166,7	144,2	0.18	1.69
Characteristics – M4.T1	252	85	950	3.10e-3	1	1.0 [s]	167,0	144,0	0.36	1.55
Characteristics – M1.T1	252	510	950	5.26e-4	1	15.9 [s]	167,9	144,2	0.90	1.69
Characteristics – M6.T1	252	83	950	1.05e-3	1	1.3 [s]	142,2	139,7	-14.5	-1.48



[Figure 11: Pressure by Runge-Kutta method (a, b, c); Pressure by method of Characteristics (d, e, f).]



[Figure 12: a) Overspeed; b) Closing Time x Maximum Overpressure and Maximum Overspeed.]

To finalize this section the main observations about the capacity and performance of the methods RKM and MOC to estimate numerically the hydraulic transient at penstock after a sudden load rejection of a hydraulic machine are summarized in Table 4.

***[Table 4]:* Main observations of the results.**

Runge-Kutta – RKM	Method of Characterists - MOC
Small relative error for all meshes used	Small relative error only for uniform spatial discretizing
Excellent accuracy in the stationary part of the pressure to discretizing with uniform elements or nonuniforms	Excellent accuracy in the stationary and oscillatory parts of the pressure to discretizations with unitary Courant and uniform spatial elements
The representation of the oscillatory part of the pressure due to the return of the waves depends severely on the time step to avoid masking	The representation of the oscillatory part of the pressure due to the return of the waves is satisfactory to all used space-time meshes
Greater simulation time	Smaller simulation time
The damping of the oscillations at the end of the closing is best represented	The damping of the pressure oscillations at the end of the closing is unsatisfactorily represented
Runge-Kutta – RKM	**Method of Characteristics - MOC**
More complex numerically to be implemented due to the construction of the matrices	Easier to be implemented numerically
Non presented instability for any of used space-time meshes	Requires unitary Courant to avoid dispersion and numerical instability

7. CONCLUSION

The solution of the transient flow in a load rejection at a hydraulic turbine by Runge-Kutta method presents satisfactory accuracy and capacity to enough reproduces the phenomenon in an attractive numerical processing time for a suitable mesh. So, about these aspects, it configures as an alternative to the Method of Characteristics, with points advantageous and others disadvantaged.

The equating used at the MOC allowed apply it for the pipeline with variable diameter but without the occurrence of numerical instability.

The overpressure and overspeed for a given rejection are directly affected by the distributor closing time law and the numerical simulation of these phenomena in hydraulic transients allows assessing in the design phase the closing time law and rotational inertia required to meet the contracted values for the same.

The application of computational simulation of hydraulic transient and overspeed in existing generating units allows evaluating the potential for optimization of these parameters by changing the closing time law, with benefits for their operational safety and service life and avoiding perform such study in an experimental way.

8. NOMENCLATURE

Term	*Symbol*	*Unit*
Cross section of pipe	A	m^2
Distributor free static area	Aw	m^2
Distributor free dynamic area	At	m^2
Capacitance	C	m^3
Número de Courant	Cr	-
Diameter	D	m
Piezometric head	H	mca
Net head	Hn	mca
Moment of inertia	J	kg m^2
Inductance	L	s^2/m^2
Rotation	N	rpm
Power	P	W
Discharge	Q	m^3/s
Resistance	R	s
Vane opening	S	m
Torque	T	Nm
Number of vanes	Z	-
Wave celerity	a	m/s
Vane height	b	m
Friction factor	f	-
Acceleration of gravity	g	m/s^2
Length	l	m
Number of elements	n	-
Time	t	S
Longitudinal position	X	M
Turbine efficiency	H	-
Specific weight of water	g	N/m^3
Time step	Δt	s
Lenght of spatial elements	Δx	m

9. ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to the ITAIPU BINACIONAL for providing the data and information necessary and to the Center for Advanced Studies on Safety of Dams – CEASB belonging to the Foundation Technological Park Itaipu – FPTI for the support to this work.

10. REFERENCES

[1] IEC 60193. Hydraulic turbines, storage pumps and pump-turbines – Model acceptance tests. International Eletrotechnical Commision, 1999.

[2] Chaudry, M. H. Applied Hydraulic Transients. Springer, 3rd edition, 2014.

[3] Streeter, V.; Wylie, E. Fluid Transients. McGraw-Hill Inc, 1978.

[4] Nicolet, C. Hydroacoustic Modelling And Numerical Simulation Of Unsteady Operation Of Hydroelectric Systems. PhD thesis 3751, EPFL, Lausanne, Switzerland, 2007.

[5] Sarra, S. A. The Method of Characteristics with applications to Conservation Laws. Journal of Online Mathematics and its Applications, Vol. 3, 2003.

[6] Urroz, G. E. Hydraulic pipe transients by the the method of characteristics. 20 Mar. 2015. <http://twixar.me/0H5>, 2005.

[7] Tullis, J. P. Hydraulics of Pipelines – Pumps, Valves, Cavitation, Transients. John Wiley & Sons. New York, 1989.

[8] ITAIPU. Itaipu Hydroelectric Project: Engineering Features. Itaipu Binacional, Curitiba – PR, 1994.

[9] Iliev, V; Popvski, P; Markov, Z. Transient Phenomena Analysis in Hydroelectric Power Plants at Off-Design Operating Conditions. International Journal of Engineering Research and Applications. Vol. 2 (6), 2012.