

ARC FAULT TOLERANCE OF CARTESIAN PRODUCT OF REGULAR DIGRAPHS ON SUPER-RESTRICTED ARC-CONNECTIVITY

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Abstract

Let $D = (V(D), A(D))$ be a strongly connected digraph. An arc set $S \subseteq A(D)$ is a restricted arc-cut of D if $D - S$ has a non-trivial strong component D_1 such that $D - V(D_1)$ contains an arc. The restricted arc-connectivity $\lambda'(D)$ is the minimum cardinality over all restricted arc-cuts of D . In [C. Balbuena, P. García-Vázquez, A. Hansberg and L.P. Montejano, *On the super-restricted arc-connectivity of s-geodetic digraphs*, Networks **61** (2013) 20–28], Balbuena *et al.* introduced the concept of super- λ' digraphs. In this paper, we first introduce the concept of the arc fault tolerance of a digraph D on the super- λ' property. We define a super- λ' digraph D to be m -super- λ' if $D - S$ is still super- λ' for any $S \subseteq A(D)$ with $|S| \leq m$. The maximum value of such m , denoted by $S_{\lambda'}(D)$, is said to be the arc fault tolerance of D on the super- λ' property. $S_{\lambda'}(D)$ is an index to measure the reliability of networks. Next we provide a necessary and sufficient condition for the Cartesian product of regular digraphs to be super- λ' . Finally, we give the lower and upper bounds on $S_{\lambda'}(D)$ for the Cartesian product D of regular digraphs and give an example to show that the lower and upper bounds are best possible. In particular, the exact value of $S_{\lambda'}(D)$ is obtained in special cases.

Keywords: fault tolerance, restricted arc-connectivity, super-restricted arc-connectivity, Cartesian product, regular digraph.

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