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WEAK COMPACTNESS IN THE SPACE OF OPERATOR VALUED MEASURES $M_{ba}(\Sigma, \mathcal{L}(X, Y))$ AND ITS APPLICATIONS

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Abstract

In this note we present necessary and sufficient conditions characterizing conditionally weakly compact sets in the space of (bounded linear) operator valued measures $M_{ba}(\Sigma, \mathcal{L}(X, Y))$. This generalizes a recent result of the author characterizing conditionally weakly compact subsets of the space of nuclear operator valued measures $M_{ba}(\Sigma, \mathcal{L}_1(X, Y))$. This result has interesting applications in optimization and control theory as illustrated by several examples.

Keywords: space of operator valued measures, weak compactness, semigroups of bounded linear operators, optimal structural control.

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