NBER WORKING PAPER SERIES

FLYING OR TRAPPED?

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Working Paper 27278
http://www.nber.org/papers/w27278

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2020

We are grateful for comments by Michele Boldrin, Joe Kaboski, Justin Lin, Danny Quah, Mike Sposi, Yong Wang, Yi Wen, and participants at the Fukuoka Conference, the INSE Conference, the Kobe Conference, the Midwest Macro Meetings, and the Society for Economic Dynamics Conference. Financial support from Academia Sinica, Kobe University, Peking University, JSPS Grant-in-Aid for Scientific Research (Nos. 15H05729, 16H03598, 17H02524, 20K01647), and the Weidenbaum Center of Washington University to enable this international collaboration is gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 27278
May 2020
JEL No. D2,E2,O4

ABSTRACT

We develop a unified theory with endogenous technology choice in human/knowledge capital accumulation to establish a rich array of equilibrium development paradigms, including poverty trap, middle income trap and flying geese growth. We then generalize the baseline structure and establish conditions for different development paradigms to arise. By calibrating the general model to fit the data from several representative economies with different income and growth patterns, we identify various prolonged flying geese episodes and middle income traps. By performing growth accounting, we find that improving human capital accumulation efficacy and mitigating barriers to human capital accumulation are most rewarding for advancing the economy and avoiding the middle income trap.

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1 Introduction

During the post WWII era, one has witnessed a widening world income disparity with the per capita real income ratio of the richest to poorest 10% rising from below 20 in 1960 to above 50 since the turn of the new millennium. This further promotes the study of poverty trap to better understand why the poorest countries failed to advance. Not until recently, more development economists have recognized the presence of broadly defined "middle income trap" in which many previously fast growing middle income and even some more advanced countries have suffered sluggish growth. This is not only a concern to those trapped countries but a worldwide issue because many of them have been primary forces for advancing global growth.

To motivate our study, we show cross-country income mobilities over the past four decades. In Table 1, the proportion of countries transiting from the ith quintile (denoted Qi) in year t to the jth quintile (Qj) in year $t+20$ over two thirty-year windows (1971-2001 and 1981-2011). The high persistence in Q1-Q1 transition is consistent with the conventional argument of poverty trap. Moreover, countries in Q3 and Q4 face high probabilities, ranging from 15 to 30%, to fall back by a quintile. During 1981-2011, in particular, such a probability for countries in Q4 to fall back is particularly high (28%); even for those in Q2 such a probability also exceeds 20%. This observation is consistent with the possibility of middle income trap. More systematic empirical evidence for the existence of middle income trap has been provided by Eichengreen, Park and Shin (2013) among others. In their work, middle income trap is identified as a substantive fall (2% or more) in per capita real income growth of a previously fast growing (3.5% or higher) middle income country (with per capita real income exceeding US$10,000 in 2005 constant PPP prices) for a considerable duration (7 year before and after the structural break). Under these criteria in conjunction with Chow tests, they find many such traps in Asian, European and Latin American countries in different years depending on their stages of development.[1] More interestingly, upon checking a number of possible drivers,

[1]Cases identified include: (i) in the 1970s, Greece (1972), Finland (1974), Japan (1974), Venezuela

they find that more educated at secondary and higher education levels is a robust factor leading to lower likelihood for a country to fall into the middle income trap.

Table 1. World-Income Mobility Matrix

		window: 1971-2001				
		$t+20$				
		Q1	Q2	Q3	Q4	Q5
	Q1	0.814	0.156	0.030	0	0
	Q2	0.114	0.650	0.191	0.045	0
t	Q3	0.082	0.164	0.559	0.168	0.027
	Q4	0	0.027	0.205	0.432	0.336
	Q5	0	0	0.048	0.290	0.662
		window: 1981-2011				
		$t+20$				
		Q1	Q2	Q3	Q4	Q5
	Q1	0.784	0.182	0.009	0.019	0.006
	Q2	0.201	0.649	0.146	0.003	0
t	Q3	0.023	0.159	0.571	0.247	0
	Q4	0	0	0.282	0.529	0.188
	Q5	0	0	0	0.188	0.812

Notes. The proportion of countries transiting from the ith quintile in year t to the jth quintile in year $t+20$ is presented over two thirty-year windows.

Two natural questions arise. First, can one establish a unified theory under which not only a poverty trap but a middle income trap may also exist? Second, can human capital, or, more generally knowledge capital, play a key role affecting the presence of the middle income trap to support the aforementioned empirical finding? In this paper, we will address both questions within an optimal growth framework with endogenous technology choice. More specifically, we consider a representative agent to maximize her lifetime utility subject to periodic budget constraints that allocate income to consumption and investments in physical and human or knowledge capital. While investment in physical capital directly contribute to its stock one for one, investment in human

(1974), Ireland (1978), (ii) in the 1980s, Singapore (1980), Mexico (1981), Puerto Rico (1988), Cyprus (1989), Korea (1989), and, (iii) in the 1990s, Portugal (1990), Hong Kong (1993), Taiwan (1995).

capital depends crucially on the knowledge accumulation technology. The knowledge accumulation technology exhibits an important feature: while better technology is more productive, it is associated with higher scale barrier. As a result of this trade-off, only those with higher existing stock of human capital may adopt better technology.

In this otherwise simple growth model, we are able to establish conditions under which a poverty trap and at least one middle income trap coexist. We find that a middle income trap is more likely to arise when the productivity of the prevailing technology is not too large, the scale barrier of the prevailing technology is sufficiently high and the productivity jump to the better technology is sufficiently big. But what happens if an economy is not mired in such middle income trap? We establish the conditions under which the economy features a flying geese development paradigm à la Akamatsu (1962), with continual technology upgrading in human/knowledge accumulation over time. We also show that with negligible productivity upgrading an economy can be permanently trap in poverty and that with negligible increase in technology scale barrier an economy can grow rapidly into an advanced society.

We then generalize our baseline model to permit employment variations over time and to relax the ranking assumption on the knowledge capital accumulation technology. By applying our modified conditions for flying geese paradigm and middle income trap to a representative set of 14 countries/economies using data from Penn World Table (PWT9.0), we find that even in the four developed countries and four fast growing Asian Tigers where they have experienced flying geese at prolonged periods, there still exist occasions of traps. In the three emerging growing economies, such traps become more often, but still less frequent than flying geese. In the three development laggards featuring chopped and slower growth paths, on the contrary, they are trapped as much as flying. Although with different sample of countries, we are able to compare our findings with those in the literature by focusing on common samples. We find that several, but not all, traps identified in our paper are in line with those in previous studies.

Overall, the results suggest that large drops in human capital technology efficacy are overwhelmingly the primary force for a country to fall into a middle income trap. Large increases in barriers to human capital accumulation are also important, playing a particularly bigger role in emerging growing countries. By contrast, total factor productivity TFP slowdowns are only essential occasionally. By performing growth accounting, we find that human capital technology upgrading and human capital barrier reduction are crucial, contributing to 37-55% of economic growth on average. In contrast to conventional growth accounting studies, TFP turns out to be largely inconsequential. Notably, in more than half of the episodes considered in our study (15 out of 28 episodes in 14 countries), human capital technology upgrading and human capital barrier reduction jointly account for more than 50% of economic growth. While human capital technology upgrading is more important in advanced and fast-growing countries, human capital barrier reduction is more crucial for emerging growing countries and development laggards.

The main message delivered by this paper is that human capital upgrading and barrier reduction can play a key role in economic development, determining whether a country may be flying or trapped. Our quantitative analysis yields an important policy implication: public policy toward improving human or knowledge capital accumulation efficacy or mitigating barriers to human or knowledge capital accumulation is likely more rewarding than that directly toward advancing the TFP.

Literature

There is a sizable literature showing the existence of a poverty trap. The argument is based on multiple steady states: coexistence of degenerate low equilibrium (trap) and a high equilibrium (see a comprehensive survey by Azariadis and Stachurski 2005). By limiting our attention to human capital related studies, poverty trap may arise due to human capital threshold externality (Azariadis and Drazen 1990; Redding 1996), moral hazard problem associated with human capital investment (Tssidon 1992), or barrier to investment in children's human capital (Galor and Weil 2000). Yet their frameworks cannot be used to generate middle income trap.

There is a small body of research on the flying geese paradigm with micro-founded theory. In particular, the flying geese paradigm may arise as a result of product upgrading (Matsuyama 2002), industry upgrading (Wang and Xie 2004; Ju, Lin and Wang 2015) or staged assimilation of global technologies (Wang, Wong and Yip 2018). None of these papers fully characterizes the local and global dynamics of the model.

In Wang, Wong and Yip (2018), the possibility of middle income trap via assimilation of global technologies is established. Specifically, technology assimilation in a country is more effective if its factor endowment gap from the technology source country is small. When a country accumulates the disadvantage capital to reduce such a gap in the process of assimilation, the mismatch is mitigated and output grows faster. But a country may over-accumulate eventually, causing more serious mismatch and growth slowdown and yielding a middle income trap. By contrast, not only we propose a very different mechanism, but we also fully establish the explicit conditions for a middle income trap and fully characterize the dynamics to ensure stability property of such a trap.

2 The Model

Consider a discrete-time model with time indexed by t. The economy consists of identical infinitely lived agents. There is a single general good that can be used for consumption or investment purposes. In addition to labor, there are two capital inputs: physical and human capital. The key feature of the model is that the human capital updating technology is endogenously determined by discrete choice with costly barriers.

2.1 The environment

A representative agent at period t produces general goods Y_t by applying a Cobb-Douglas production technology $Y_t = AH_{t-1}^{\alpha}K_{t-1}^{\beta}L_t^{\gamma}$ ($\alpha, \beta, \gamma \in (0, 1)$, $\alpha + \beta + \gamma = 1$), the inputs of which are physical capital K_{t-1}, labor L_t, and human capital H_{t-1} – which should be taken more generally to include knowledge capital and know-how. Note that the production of general goods takes one gestation period in which the human capital, H_{t-1}, and the physical capital, K_{t-1}, are prepared one period before the production occurs (time-to-build). A is the total factor productivity (TFP) of the production technology.[2] The general goods are consumed or used for investments by the representative agent. The aggregate budget constraint is given by

$$AH_{t-1}^{\alpha}K_{t-1}^{\beta}L_t^{\gamma} = C_t + I_t^h + I_t^k, \tag{1}$$

where C_t is aggregate consumption, and I_t^h and I_t^k are investments for production of human capital and physical capital, respectively. The per worker budget constraint is obtained from Eq. (1) as follows:

$$y_t := Ah_{t-1}^{\alpha}k_{t-1}^{\beta} = c_t + i_t^h + i_t^k, \tag{2}$$

where $h_{t-1} := H_{t-1}/L_t$, $k_{t-1} := K_{t-1}/L_t$, $c_t := C_t/L_t$, $i_t^h := I_t^h/L_t$, and $i_t^k := I_t^k/L_t$ are per worker variables. Although the growth in labor force is introduced in the quantitative analysis in section 5, the population of labor force is normalized to $L_t = 1$ for all $t \geq 0$ in the theoretical analysis up to section 4.

2.2 Optimization Problem

Let us first specify a representative agent's optimization problem by maximizing her lifetime utility subject to three constraints in addition to the typical nonnegativity constraints on investments:

$$\max \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_{\tau}$$

[2] In the quantitative analysis in section 5 where we generalize the basic model, A is allowed to be time-variant.

subject to

$$
\begin{aligned}
y_\tau \quad &: \quad = A h_{\tau-1}^{\alpha} k_{\tau-1}^{\beta} = c_\tau + i_\tau^h + i_\tau^k && (3) \\
k_\tau \quad &= \quad g_0(i_\tau^k) && (4) \\
h_\tau \quad &= \quad \max_{m=1,2,\ldots,M} \{ g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau) \}, && (5)
\end{aligned}
$$

for $\tau \geq t$, where $\delta \in (0, 1)$ is the subjective discount factor. In Eqs. (3)-(5), both physical and human capital depreciates entirely in one period. The representative agent is endowed with two types of investment projects: one is to produce physical capital and the other is to produce human capital. In the first investment project, a one-for-one simple linear technology produces capital from general goods. That is, in Eq. (4), the production function for physical capital, $g(i_\tau^k)$, is given by

$$g_0(i_\tau^k) = i_\tau^k. \tag{6}$$

In Eq. (5), $g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)$ is a production function for human capital when the representative agent applies the mth technology. Human capital is produced from the general goods where the human capital formation is subject to technology choice: the representative agent chooses the best technology among a number of M technologies for the human capital formation.

Throughout the analysis, we shall begin by analyzing the case in which there are three technologies, subsequently followed by generalization with M technologies. In each period, the agent chooses the best technology for human (or knowledge) capital formation and solves her maximization problem, exogenously given the externalities from the (average) past human capital, $\bar{h}_{\tau-1}$, and the (average) current-period output, $\bar{y}_\tau$, in the economy. We impose Assumption 1 below for regulating the functional form of $g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)$.

Assumption 1. *(i)* $\partial g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i_\tau^h > 0$ *(ii)* $\partial^2 g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i_\tau^h \partial \bar{h}_{\tau-1} > 0$, *(iii)*

$\partial^3 g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial i^h_\tau \partial \bar{h}^2_{\tau-1} < 0$, *and (iv)* $\partial^2 g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau)/\partial \bar{h}_{\tau-1}\partial \bar{y}_\tau < 0$.

Assumption 1-(i) guarantees the positive marginal product of human capital investment, i^h_τ. Assumption 1-(ii) implies that the past knowledge accumulation, which is condensed into the past human capital formation $\bar{h}_{\tau-1}$, has a positive external effect on the marginal product promoting the human capital formation. However, from Assumption 1-(iii), the effect of the positive externality diminishes as the past knowledge more accumulates. Assumption 1-(iv) mitigates the scale effect of human capital externality. Since as the economy develops, $\bar{h}_{\tau-1}$ and $\bar{y}_\tau$ continue to rise, so this assumption limits the scale of knowledge spillovers.

Under Assumption 1, the representative agent chooses the best technology depending upon the human capital accumulation. If the productivity with respect to a certain technology becomes small, another suitable technology may be chosen as noted from the max operator in the right-hand side of Eq. (5).

To make the following analysis concrete, we specify $g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau)$ as

$$g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau) = \frac{B_m(\bar{h}_{\tau-1})}{\bar{y}_\tau} i^h_\tau, \tag{7}$$

where $B_m(\bar{h}_{\tau-1}) := \theta_m(\bar{h}_{\tau-1} - \eta_m)^\sigma$ for $\bar{h}_{\tau-1} \geq \eta_m$, with $\alpha < \sigma \in (0,1)$, $\theta_m \in [1, \infty)$, and $\eta_m \in [0, \infty)$.[3] Thus, human or knowledge capital accumulation depends on the society's existing stock in the spirit of the knowledge spillovers in Romer (1986) and human capital spillovers in Lucas (1988). However, the externality of $\bar{h}_{\tau-1}$ is effective only when it exceeds a certain border, η_m. Thus, one may view the presence of η_m as a result of scale barrier in knowledge accumulation. The scale barrier in human capital considered here also captures the argument in Buera and Kaboski (2012) where higher skill is required for production of goods with greater complexity. Our scale barrier setup generates similar implication to the appropriate technology model in Caselli and Coleman (2006) where skilled labor abundant rich countries tend to choose technologies

[3]The assumption $\alpha < \sigma$ guarantees the net effect of $\bar{h}_{\tau-1}$ is always positive.

more efficient to skilled workers. As imposed a parameter condition for $B_m(\bar{h}_{\tau-1})$ in Assumption 2 below, there is a trade-off between the productivity, θ_m, and the scale barrier, η_m.

Assumption 2. *(i)* $1 = \theta_1 < \theta_2 < \cdots < \theta_M$ *and (ii)* $0 = \eta_1 < \eta_2 < \cdots < \eta_M$.

Assumption 2 implies that human capital accumulation needs to exceed the higher border for an economy to utilize the higher productivity technology. It may be thought of as after the highest technology M, the economy will be on a perpetual balanced growth path. That is, along such a balanced growth path, human capital production is simply linear with $B_{m>M}(\bar{h}_{\tau-1}) := \theta_{\max}\bar{h}_{\tau-1}$ for $\bar{h}_{\tau-1} > \eta_{\max}$ where $\theta_{\max} > \theta_M$ and $\eta_{\max} > \eta_M$. This highest stage of development corresponds to the Rostovian state of Mass Consumption. Because this equilibrium is trivial, we, for the sake of brevity, will not characterize it explicitly.

The Lagrangian for the utility maximization problem is set up as follows:

$$\begin{aligned}\mathcal{L}_t := \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_\tau + \sum_{\tau=t}^{\infty} \delta^{\tau-t} \lambda_\tau \left[A h_{\tau-1}^{\alpha} k_{\tau-1}^{\beta} - i_\tau^h - i_\tau^k - c_\tau \right] \\ + \sum_{\tau=t}^{\infty} \delta^{\tau-t} p_\tau^h \left[b(\bar{h}_{\tau-1}, \bar{y}_\tau) i_\tau^h - h_\tau \right] + \sum_{\tau=t}^{\infty} \delta^{\tau-t} p_\tau^k \left[i_\tau^k - k_\tau \right],\end{aligned}$$

where $b(\bar{h}_{\tau-1}, \bar{y}_\tau) := \max_m\{B_m(\bar{h}_{\tau-1})\}/\bar{y}_\tau$, and λ_τ, p_τ^h, and p_τ^k are the shadow prices of general goods, human capital, and physical capital, respectively. The first-order conditions are given by

$$\lambda_t = \frac{1}{c_t} \tag{8}$$

$$\lambda_t = \left(\frac{\delta \alpha b(\bar{h}_{t-1}, \bar{y}_t) y_{t+1}}{h_t} \right) \lambda_{t+1} \tag{9}$$

$$\lambda_t = \left(\frac{\delta \beta y_{t+1}}{k_t} \right) \lambda_{t+1} \tag{10}$$

$$\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t) p_t^h. \tag{11}$$

The necessary and sufficient conditions for the optimality of this maximization problem consist of Eqs. (8)-(11) as well as the transversality conditions, $\lim_{t\to\infty}\delta^t p_t^k k_t = \lim_{t\to\infty}\delta^t p_t^h h_t = 0$.

3 Equilibrium

We are now prepared to define and establish the dynamic competitive equilibrium and to characterize the dynamics. A key strategy is to reduce the dynamical system to the two states – physical and human capital – and the values of these two capitals. This strategy greatly simplifies the analysis because the two capital values are proportional and their limits are governed by the transversality conditions.

From Eqs. (9) and (10), it follows that $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$. From this equation and Eqs. (4)-(7), we obtain $i_t^k + i_t^h = k_t(1-\gamma)/\beta$. These two equations allow us to rewrite the flow budget constraint (3) as

$$c_t + \frac{1-\gamma}{\beta}k_t = A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^{\alpha} k_{t-1}^{1-\gamma}, \tag{12}$$

for all $t \geq 1$. Additionally, substituting $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$ into Eq. (9) yields

$$\lambda_{t-1} = \lambda_t \delta\alpha A b(\bar{h}_{t-2}, \bar{y}_{t-1})^{\alpha}\left(\frac{\alpha}{\beta}\right)^{\alpha-1} k_{t-1}^{-\gamma}. \tag{13}$$

Multiplying λ_t to both sides of Eq. (12) leads to

$$\lambda_t c_t + \frac{1-\gamma}{\beta}\lambda_t k_t = \lambda_t A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^{\alpha} k_{t-1}^{1-\gamma}. \tag{14}$$

Applying Eqs. (8) and (13) to the left-hand and right-hand sides of Eq. (14) respectively, we obtain

$$q_t^k = \frac{1}{\delta(1-\gamma)}q_{t-1}^k - \frac{\beta}{1-\gamma}, \tag{15}$$

where $q_t^k := p_t^k k_t = \lambda_t k_t$, which is the value of physical capital in period t. It follows from Eq. (11) and $k_t = (\beta/(\alpha b(\bar{h}_{t-1}, \bar{y}_t)))h_t$ that $q_t^k = p_t^k k_t = (\beta/\alpha)p_t^h h_t$. Substituting

$q_t^k = p_t^k k_t = (\beta/\alpha) p_t^h h_t$ into Eq (15) yields

$$q_t^h = \frac{1}{\delta(1-\gamma)} q_{t-1}^h - \frac{\alpha}{1-\gamma}, \tag{16}$$

where $q_t^h := p_t^h h_t$, which is the value of human capital in period t. Eqs. (15) and (16) govern the dynamics of the values of the two capitals recursively, independent of other states or controls or technology choice.

From Eqs. (10) and (11), it follows that $p_t^k y_t = q_{t-1}^k/(\delta\beta)$. The use of this equation, Eqs. (5), (7) and (15) with $i_t^h = (\alpha/\beta) k_t$ yields

$$h_t = \alpha \max_m \{B_m(h_{t-1})\} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right], \tag{17}$$

where we have used the fact that $\bar{h}_{t-1} = h_{t-1}$ in equilibrium. Additionally, from $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta) k_t$ and Eq. (17), we obtain

$$k_t = \beta A h_{t-1}^{\alpha} k_{t-1}^{\beta} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right]. \tag{18}$$

We can now define a dynamic competitive equilibrium as a sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, for $t \geq 0$ that satisfies Eqs. (15)-(18) and the transversality condition, given $h_0 \geq 0$ and $k_0 \geq 0$. In so far, we have, however, not yet analyzed the technology choice, $\max_m\{B_m(h_{t-1})\}$, in Eq. (17), to which we now turn.

3.1 Technology choice

We now consider technology choice $\max_m\{B_m(h_{t-1})\}$ in Eq. (17). Given the trade-off under Assumption 2, the key is to determine the cutoff human capital scales between each pair of technologies. Define such cutoffs as v_1 and v_2 so that $B_1(v_1) = B_2(v_1)$ and $B_2(v_2) = B_3(v_2)$. Note that v_1 is the cutoff of h between the first and the second technologies whereas v_2 is the cutoff between the second and the third technologies. It is straightforward to show that v_1 and v_2 satisfy, respectively,

$$v_1^\sigma = \theta_2(v_1 - \eta_2)^\sigma \tag{19}$$

$$\theta_2(v_2 - \eta_2)^\sigma = \theta_3(v_2 - \eta_3)^\sigma, \tag{20}$$

from which v_1 and v_2 are uniquely determined as $v_1 = \theta_2^{\frac{1}{\sigma}}\eta_2/(\theta_2^{\frac{1}{\sigma}} - 1)$ and $v_2 = (\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2)/(\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}})$.

Lemma 1. *Under Assumption 2, the following hold.*

- *If $\eta_2 \leq h_{t-1} < v_1$ (resp. $h_{t-1} > v_1$), it holds that $B_1(h_{t-1}) > B_2(h_{t-1})$ (resp. $B_1(h_{t-1}) < B_2(h_{t-1})$).*
- *If $\eta_3 \leq h_{t-1} < v_2$ (resp. $h_{t-1} > v_2$), it holds that $B_2(h_{t-1}) > B_3(h_{t-1})$ (resp. $B_2(h_{t-1}) < B_3(h_{t-1})$).*

Proof. See the Appendix.

Proposition 1. *Suppose that Assumption 2 holds. Suppose also that $v_1 < v_2$. Then, in $\max_m\{B_m(h_{t-1})\}$, the representative agent optimally chooses the first technology ($m = 1$) if $0 \leq h_{t-1} < v_1$, the second technology ($m = 2$) if $v_1 < h_{t-1} < v_2$, and the third technology ($m = 3$) if $v_2 < h_{t-1}$.*

Proof. See the Appendix.

Note that if $h_{t-1} = v_1$, the choice between the first and second technologies is indifferent. In this case, it is assumed that the second technology is chosen over the first technology. Likewise, if $h_{t-1} = v_2$, the choice between the second and third technologies is indifferent and it is assumed that the third technology is chosen over the second technology in this case. It is clear that Proposition 1 can be readily generalized to M technologies. Suppose that $v_1 < v_2 < \cdots < v_{m-1}$ where $v_i = (\theta_{i+1}^{\frac{1}{\sigma}}\eta_{i+1} - \theta_i^{\frac{1}{\sigma}}\eta_i)/(\theta_{i+1}^{\frac{1}{\sigma}} - \theta_i^{\frac{1}{\sigma}})$. Then, the representative agent optimally chooses the first technology if $0 \leq h_{t-1} < v_1$, the

mth technology if $v_{m-1} \leq h_{t-1} < v_m$ ($m = 2, ...M-1$), and the Mth technology if $v_{M-1} \leq h_{t-1}$.

3.2 Steady states

The steady states can be solved in a recursive manner by solving the capital values first. Eqs. (15)-(18) form the dynamical system in equilibrium, which we recite in the following:

$$\begin{cases} q_t^h = \frac{1}{\delta(1-\gamma)} q_{t-1}^h - \frac{\alpha}{1-\gamma} \\ q_t^k = \frac{1}{\delta(1-\gamma)} q_{t-1}^k - \frac{\beta}{1-\gamma} \\ h_t = \alpha \max_m \{B_m(h_{t-1})\} \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right] =: J_1(q_{t-1}^k, h_{t-1}, k_{t-1}) \\ k_t = \beta A h_{t-1}^\alpha k_{t-1}^\beta \left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma} \left(\frac{1}{q_{t-1}^k} \right) \right] =: J_2(q_{t-1}^k, h_{t-1}, k_{t-1}). \end{cases} \tag{21}$$

As a result of the recursive property mentioned above, the values of human and physical capital in the steady state, q^{h*} and q^{k*}, are independent of human and physical capital and the technology choice

$$q^{h*} = \frac{\alpha\delta}{1-\delta(1-\gamma)} \text{ and } q^{k*} = \frac{\beta\delta}{1-\delta(1-\gamma)}. \tag{22}$$

In contrast, physical capital and human capital in the steady state depend crucially on the technology choice. Suppose that the jth technology in human capital production is optimally chosen, i.e., $\max_m\{B_m(h_{t-1})\} = B_j(h_{t-1})$ ($j = 1, 2$ or 3). Since $B_j(h_{t-1})$ is concave, we can "potentially" obtain at most two steady states, say, $h_{j,1}^*$ and $h_{j,2}^*$, from the technology. From (21), it follows that

$$h_{j,s}^* = \alpha\delta\theta_j (h_{j,s}^* - \eta_j)^\sigma \tag{23}$$

$$k_{j,s}^* = \beta\delta A (h_{j,s}^*)^\alpha (k_{j,s}^*)^\beta, \tag{24}$$

for $s = 1, 2$. Assuming that $h_{j,1}^* < h_{j,2}^*$, we call $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$ and $(q^{h*}, q^{k*}, h_{j,2}^*, k_{j,2}^*)$

the low steady state and the high steady state, respectively.

From Eq. (23), the potential two steady states of human capital, $h^*_{j,1}$ and $h^*_{j,2}$, satisfy the following equation:

$$\Pi_j(h) := h^{\frac{1}{\sigma}} - (\delta\alpha\theta_j)^{\frac{1}{\sigma}} h + (\delta\alpha\theta_j)^{\frac{1}{\sigma}} \eta_j = 0. \tag{25}$$

Because $\Pi_j(h)$ is convex and $\hat{h}_j := \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}$ gives a minimum of $\Pi_j(h)$, there exist two distinct real number solutions of Eq. (25) if and only if $\Pi_j(\hat{h}_j) < 0$. Formally, we have Lemma 2 below.

Lemma 2. *There exist two distinct real number solutions of Eq. (25) if and only if*

$$\eta_j - \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}(1-\sigma) < 0. \tag{26}$$

Proof. The claim of Lemma 2 follows from the fact that $\Pi_j(h)$ is convex, $\Pi_j(0) > 0$, and $\Pi_j(\hat{h}) < 0 \iff$ Eq. (26). $\square$

In what follows, we derive conditions that each technology actually has two steady states (including the trivial one) in the dynamical system.

3.2.1 First technology: $j = 1$

Note that under Assumption 2, Eq. (25) with $j = 1$ has two solutions, $h^*_{1,1} = 0$ and

$$h^*_{1,2} := (\delta\alpha)^{\frac{1}{1-\sigma}}, \tag{27}$$

From Eq. (24), we have

$$k^*_{1,2} := (\delta\beta A)^{\frac{1}{1-\beta}}(h^*_{1,2})^{\frac{\alpha}{1-\beta}}. \tag{28}$$

Proposition 2. *Under Assumption 2, suppose that the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2. \tag{29}$$

Then, a non-trivial steady state, $(q^{h*}, q^{k*}, h^*_{1,2}, k^*_{1,2})$, associated with the first technology exists in the dynamical system.

Proof. It suffices to show that $h^*_{1,2} < v_1$. From Eq. (29), it holds that $h^*_{1,2} < \eta_2 < v_1$, and Eq. (28) gives $k^*_{1,2}$. Then, the desired conclusion holds. □

3.2.2 Second technology: $j = 2$

We next turn to the case with $j = 2$ under which there are still two steady states prevailed.

Proposition 3. *Under Assumption 2, suppose that the following parameter conditions hold:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{30}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3. \tag{31}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{2,1}, k^*_{2,1})$ and $(q^{h*}, q^{k*}, h^*_{2,2}, k^*_{2,2})$, associated with the second technology exist in the dynamical system.

Proof. See the Appendix.

3.2.3 Third technology: $j = 3$

When the highest technology is chosen ($j = 3$), it once again features two steady states.

Proposition 4. *Under Assumption 2, suppose that the following parameter condition holds:*

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}. \tag{32}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{3,1}, k^*_{3,1})$ and $(q^{h*}, q^{k*}, h^*_{3,2}, k^*_{3,2})$, associated with the third technology exist in the dynamical system.

Proof. See the Appendix.

3.2.4 Summary

Propositions 2-4 imply that six steady states (including a trivial one) exist in the dynamical system if the following inequalities hold:

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{33}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}, \tag{34}$$

in which each technology yields two steady states.

It is straightforward to extend the analysis to the case in which the number of technologies is M with $1 = \theta_1 < \cdots < \theta_M$ and $0 = \eta_1 < \cdots < \eta_M$. In this case, $2M$ steady states appear if the following inequalities hold:

$$\begin{aligned}
&(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}}\\
&(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}\\
&\qquad\qquad\vdots\\
&(\delta\alpha\theta_{M-1})^{\frac{1}{1-\sigma}} < \eta_M < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_M)^{\frac{1}{1-\sigma}}.
\end{aligned}$$

3.3 Local dynamics

Suppose that the jth technology has two steady states: a lower one, $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$, and a higher one, $(q^{h*}, q^{k*}, h^*_{j,2}, k^*_{j,2})$, where $h^*_{j,1} < h^*_{j,2}$. Linearization of the dynamical system (21) with the jth technology around one of the steady states, $(q^{h*}, q^{k*}, h^*_{j,s}, k^*_{j,s})$ ($s = 1$ or 2), implies

$$\begin{pmatrix} q^h_t - q^{h*} \\ q^k_t - q^{k*} \\ h_t - h^*_{j,s} \\ k_t - k^*_{j,s} \end{pmatrix} = \begin{pmatrix} \frac{1}{\delta(1-\gamma)} & 0 & 0 & 0 \\ 0 & \frac{1}{\delta(1-\gamma)} & 0 & 0 \\ 0 & J_{1,q^k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \alpha\delta B'_j(h^*_{j,s}) & 0 \\ 0 & J_{2,q^k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & J_{2,h}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \beta \end{pmatrix} \begin{pmatrix} q^h_{t-1} - q^{h*} \\ q^k_{t-1} - q^{k*} \\ h_{t-1} - h^*_{j,s} \\ k_{t-1} - k^*_{j,s} \end{pmatrix}, \tag{35}$$

where $J_{n,q^k}(q^k, h, k) := \partial J_n(q^k, h, k)/\partial q^k$ and $J_{n,h}(\lambda, h, k) := \partial J_n(\lambda, h, k)/\partial h$ for $n = 1, 2$. The eigenvalues of this dynamical system are given by $\rho_1 := 1/(\delta(1-\gamma))$, $\rho_2 := 1/(\delta(1-\gamma))$, $\rho_3 := \alpha\delta B_j'(h_{j,s}^*)$, and $\rho_4 = \beta$.

In the dynamical system (21), whereas h_t and k_t are state variables that cannot jump being predetermined variables, q_t^h and q_t^k can jump, which are determined by expectations. Due to the recursive property, the dynamical system can be fully characterized. In particular, since $\rho_1,\ \rho_2 > 1$ and $0 < \rho_4 < 1$, the property of the local dynamics depends entirely on $\alpha\delta B_j'(h_{j,s}^*)$.

Lemma 3. *Under Assumption 2, suppose that inequalities (33) and (34) are satisfied. Then, for any j, it holds that $\alpha\delta B_j'(h_{j,1}^*) > 1$ and $0 < \alpha\delta B_j'(h_{j,2}^*) < 1$.*

Proof. See the Appendix.

It is seen from Lemma 3 that in the linearized dynamical system around the lower steady state, $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$, the three eigenvalues are greater than one and the absolute value of the one eigenvalue is less than one. Therefore, no equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, exists that converges to the lower steady state. Also from Lemma 3, one can see that in the linearized dynamical system around the higher steady state, $(q_t^h, q_t^k, h_{j,2}^*, k_{j,2}^*)$, the two eigenvalues are greater than one and the absolute values of the two eigenvalues are less than one. Therefore, a unique equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, exists around the higher steady state that converges to this steady state. We summarize these results in Proposition 5 below.

Proposition 5. *Under Assumption 2, suppose that inequalities (33) and (34) are satisfied. Consider the jth technology. Then, the following hold:*

- *There exists no equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, around the lower steady state, $(q^{h*}, q^{k*}, h_{j,1}^*, k_{j,1}^*)$, that converges to this steady state.*
- *There exists a unique equilibrium sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, around the higher steady state, $(q^{h*}, q^{k*}, h_{j,2}^*, k_{j,2}^*)$, that converges to this steady state.*

Proof. The discussion just before Proposition 5 proves the claims. □

4 Global Analysis

In the previous section, we analyzed the local dynamic property. In the analysis, it is still unclear whether an equilibrium sequence exists around the lower steady state. Moreover, even if an equilibrium sequence exists around the lower steady state, the analysis of local dynamics does not clarify where it goes. In this section, we address these questions.

4.1 Phase diagrams

In Eq. (21), the difference equations with respect to q_t^h and q_t^k, i.e., Eqs. (15) and (16), are independent of h_t and k_t, and they are solvable analytically. Solving Eqs. (15) and (16) forward, we obtain

$$q_t^k = [\delta(1-\gamma)]^u q_{t+u}^k + \beta\delta + \beta\delta[\delta(1-\gamma)] + \cdots + \beta\delta[\delta(1-\gamma)]^{u-1} \tag{36}$$

$$q_t^h = [\delta(1-\gamma)]^u q_{t+u}^h + \alpha\delta + \alpha\delta[\delta(1-\gamma)] + \cdots + \alpha\delta[\delta(1-\gamma)]^{u-1}. \tag{37}$$

It follows from the transversality condition, $\lim_{u\to\infty} \delta^u q_{t+u}^k = \lim_{u\to\infty} \delta^u q_{t+u}^h = 0$, that $\lim_{u\to\infty}[\delta(1-\gamma)]^u q_{t+u}^k = \lim_{u\to\infty}[\delta(1-\gamma)]^u q_{t+u}^h = 0$. Therefore, Eqs. (36) and (37) yield $q_t^k = \beta\delta/(1-\delta(1-\gamma))$ and $q_t^h = \alpha\delta/(1-\delta(1-\gamma))$, respectively, which implies that the equilibrium sequences of $\{q_t^k, q_t^h\}$ are uniquely determined, being equal to $\{q_t^k, q_t^h\} = \{q^{k*}, q^{h*}\}$.

Suppose that $v_1 < v_2$ with Assumption 2. Then, from Proposition 1, Eqs. (17) and (18) become

$$h_t = \begin{cases} \alpha\delta h_{t-1}^{\sigma} = \alpha\delta B_1(h_{t-1}) & \text{if } 0 \le h_{t-1} < v_1 \\ \alpha\delta\theta_2(h_{t-1}-\eta_2)^{\sigma} = \alpha\delta B_2(h_{t-1}) & \text{if } v_1 \le h_{t-1} < v_2 \\ \alpha\delta\theta_3(h_{t-1}-\eta_3)^{\sigma} = \alpha\delta B_3(h_{t-1}) & \text{if } v_2 \le h_{t-1} \end{cases} \tag{38}$$

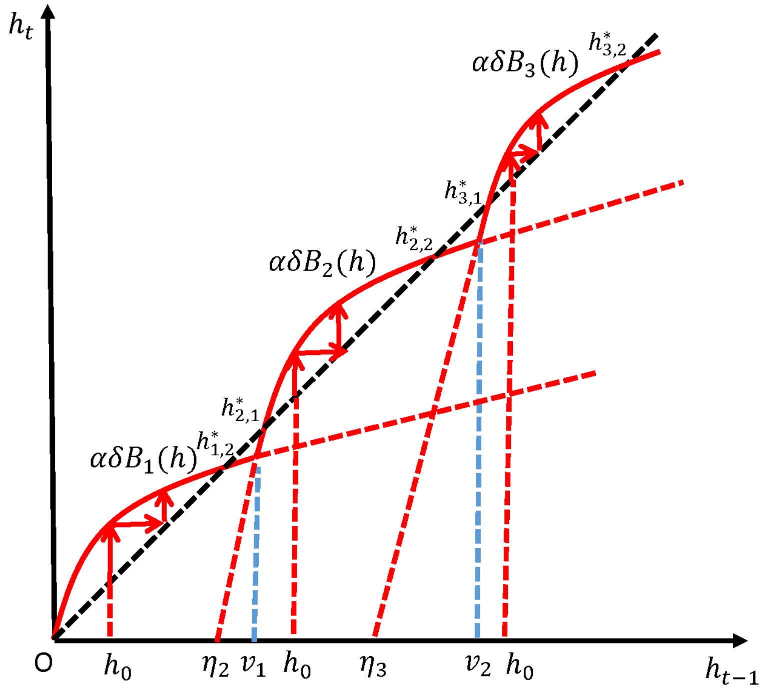

Figure 1. Phase diagram of h_t

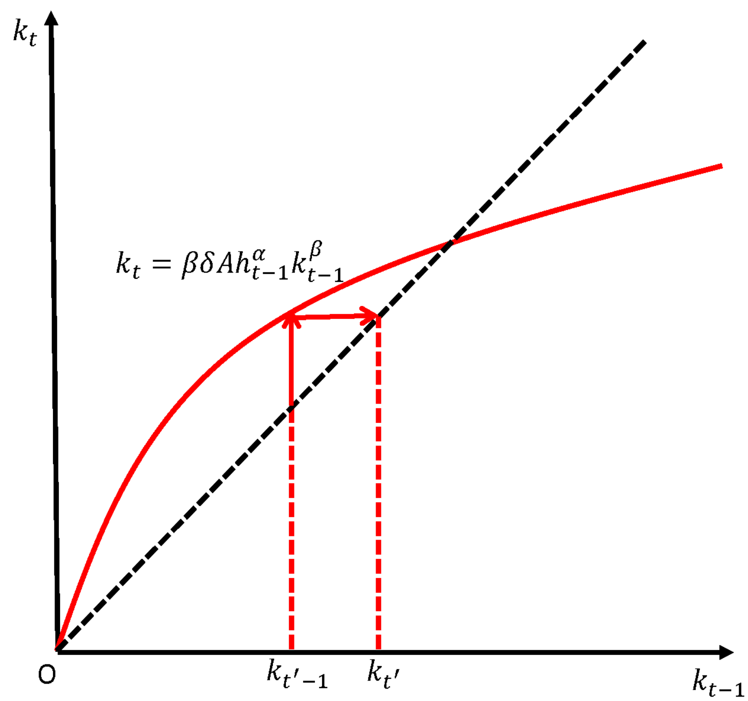

Figure 2. Conditional phase diagram of k_t given h_{t-1}

and

$$k_t = \beta\delta A h_{t-1}^{\alpha} k_{t-1}^{\beta}. \tag{39}$$

Figure 1 provides the phase diagram of Eq. (38) when inequalities (33) and (34) hold under Assumption 2 and Figure 2 provides the conditional phase diagram of Eq. (39) with h_{t-1} given.

4.2 Take-off and flying geese

If the productivity of human capital formation for each technology, $B_m(\bar{h}_{t-1})/\bar{y}_t$, is high, the economy does not fall in the low- or middle-income traps. Proposition 6 below provides a condition that the economy does not fall in the low- or middle-income traps and converges to the high steady state of the third technology even if the initial human capital is very low.

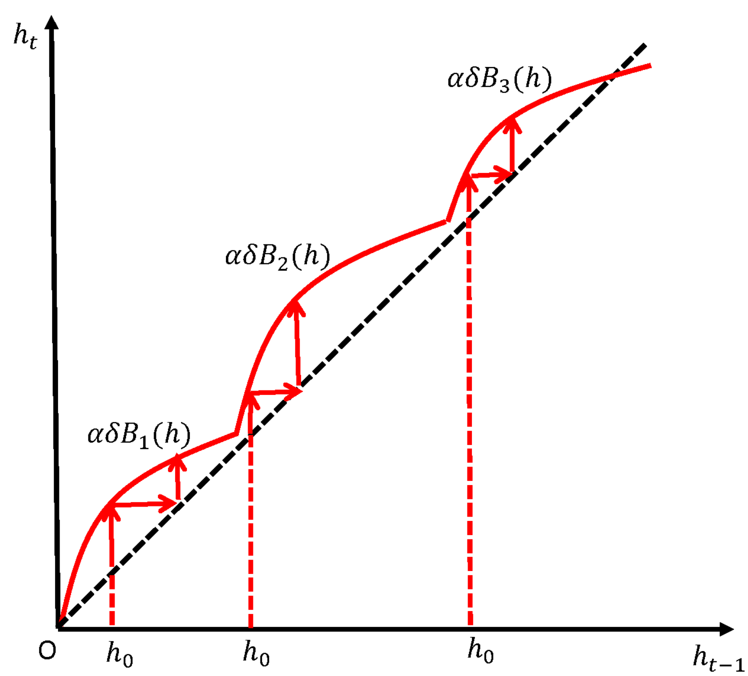

Figure 3. Flying geese pattern

Proposition 6. *Suppose that Assumption 2 holds. Then, there exist only two steady* *states in the dynamical system, which are a trivial one and the high steady state of the third technology, if the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \Phi_2\}, \tag{40}$$

where

$$\Phi_i := \frac{\left(\theta_{i+1}^{\frac{1}{\sigma}}\eta_{i+1} - \theta_i^{\frac{1}{\sigma}}\eta_i\right)^{\frac{1}{1-\sigma}}}{\theta_i^{\frac{1}{1-\sigma}}\theta_{i+1}^{\frac{1}{1-\sigma}}\left(\theta_{i+1}^{\frac{1}{\sigma}} - \theta_i^{\frac{1}{\sigma}}\right)(\eta_{i+1} - \eta_i)^{\frac{\sigma}{1-\sigma}}}.$$

Proof. See the Appendix.

Figure 3 provides the phase diagram of Eq. (38) when inequality (40) holds under Assumption 2.

The outcome of Proposition 6 can be extended to the case in which the number of technologies is M. Under Assumption 2, there exist two steady states: one is a trivial one and the other is the high steady state of the Mth technology if and only if $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$. When this inequality holds, the economy develops from a low human capital state to a high human capital state proceeding along the M technologies.

By examining this inequality, one can see that if either changes in θ_i or changes in η_i becomes negligible (but not both), then Φ_i tends to be larger and the flying geese paradigm is less likely to arise. The two polar cases are actually different. When the barrier to human capital technology upgrading is essentially unchanged, the development pattern exhibits immediate upgrading to the highest technology M as soon as the level of human capital exceeds the barrier. On the contrary, when the productivity gain from upgrading is nil, the economy is trapped in the lowest technology.

4.3 Middle income trap

What if a certain Φ_j at an intermediate technology j breaks the inequality $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$ stated in the previous subsection? In this case, the economy can fall into a trap with the jth technology. More precisely, if there is a Φ_j such that $\Phi_j > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{j-1}, \Phi_{j+1}, \ldots, \Phi_{M-1}\}$ and if $v_{j-1} < v_j < v_{j+1}$, then the economy converges to the high steady state of the jth technology if the economy starts with low human capital. This phenomenon is depicted in Figure 4.

While the trivial equilibrium is frequently referred to as the low income trap, the equilibrium at the jth technology can be called a middle income trap. In general, there can be more than one technology at which a middle income country may be trapped. To establish this main result, define the set of all globally available technologies as $T := \{1, \cdots, M\}$ and the set of "interior" technologies as $I := \{2, \cdots, M-1\}$. Generically, it is conveniently summarized below.

Proposition 7. *Suppose that Assumption 2 holds. Then, there can exist nontrivial steady states in the dynamical system, featuring middle income trap at various technologies in $J \subseteq I$ if the following parameter conditions holds:*

$$\min_{j \in J}\{\Phi_j\} > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max_{m \in T \setminus J}\{\Phi_m\}, \tag{41}$$

$$v_{i-1} < v_i \quad \textit{for all } i \in I. \tag{42}$$

Proof. See the Appendix.

We next turn to characterizing under what circumstances a middle income trap is more likely to arise. From the first inequality of (41), Φ_j of the jth technology in J satisfies the following inequality:

$$\Phi_j := \frac{\left(\theta_{j+1}^{\frac{1}{\sigma}}\eta_{j+1} - \theta_j^{\frac{1}{\sigma}}\eta_j\right)^{\frac{1}{1-\sigma}}}{\theta_j^{\frac{1}{1-\sigma}}\theta_{j+1}^{\frac{1}{1-\sigma}}\left(\theta_{j+1}^{\frac{1}{\sigma}} - \theta_j^{\frac{1}{\sigma}}\right)(\eta_{j+1} - \eta_j)^{\frac{\sigma}{1-\sigma}}} > (\delta\alpha)^{\frac{1}{1-\sigma}}. \tag{43}$$

It is straightforward to show that inequality (42) holds for $i = j, j+1$ if

$$\theta_j^{\frac{1}{\sigma}}(\eta_{j+1} - \eta_j) > \theta_{j-1}^{\frac{1}{\sigma}}\eta_{j+1} \tag{44}$$

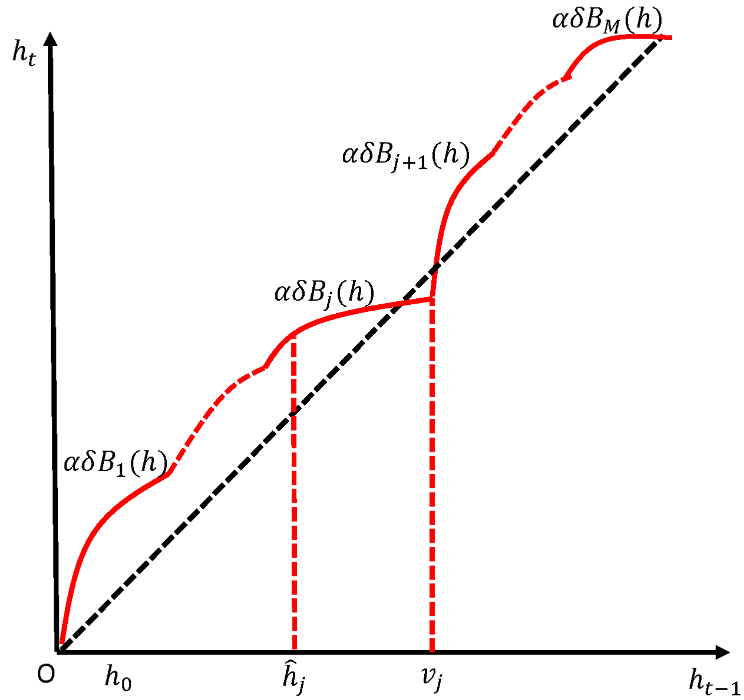

Figure 4. Trap in the jth technology

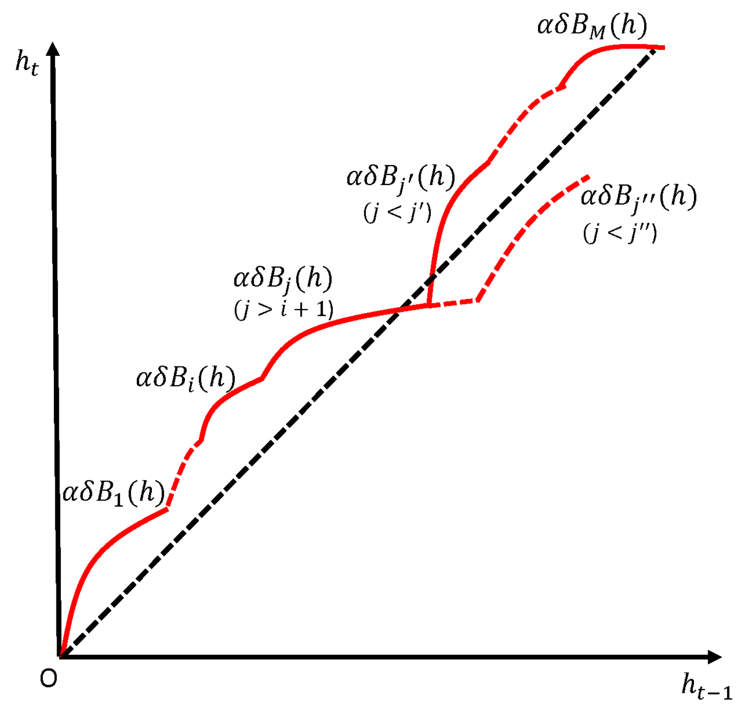

Figure 5. Restricted middle income trap

$$\theta_{j+1}^{\frac{1}{\sigma}}(\eta_{j+2}-\eta_{j+1}) > \theta_j^{\frac{1}{\sigma}}\eta_{j+2}. \tag{45}$$

Let $g_{\theta_j} = \theta_{j+1}/\theta_j$ and $g_{\eta_j} = \eta_{j+1}/\eta_j$ capture, respectively, the productivity gap and the barrier gap between the j+1th and the jth technologies under inequalities (44) and (45). Then, inequality (43) reduces to

$$\frac{(\eta_j)^{1-\sigma}\left[(g_{\theta_j})^{\frac{1}{\sigma}}g_{\eta_j}-1\right]}{\theta_j g_{\theta_j}\left[(g_{\theta_j})^{\frac{1}{\sigma}}-1\right]^{1-\sigma}\left(g_{\eta_j}-1\right)^{\sigma}} > \delta\alpha. \tag{46}$$

It is straightforward to show that the left-hand side of this inequality is strictly decreasing in θ_j and strictly increasing in η_j. Thus, other things being equal, a middle income trap of the jth technology for $1 < j < M$ is likely to arise if the productivity of the jth technology is not too large and the scale barrier of the jth technology is sufficiently high.

While we have established a sufficient condition given by (41) and (42) for broadly defined middle income trap to arise, one may inquire whether our model may support a more restricted middle income trap, such as to satisfy the conditions outlined by Eichengreen, Park and Shin (2013). We will elaborate on this issue using Figure 5. First of all, our technology choice allows for a sustained flying geese paradigm that may even feature jumps from, for example, the ith technology to the jth technology ($j > i+1$). Continual technology upgrading in conjunction with some technology leap frogging (jumps) would ensure that the country under consideration experiences fast growth, thus satisfying the growth condition (say, at least 3.5% annually). However, when the conditions stated by (41) and (42) hold at the jth technology, a country is stuck therein. When barriers to the next generations of technologies are high and the corresponding productivity gaps are big, there could be several generations of technologies not worth adopting (such as $B_{j''}$, $j < j'', j'$). As a result, the country may stay at the jth technology for years before pulling out (to the j+1th technology), thus causing relatively low growth (say, at least 2% lower than the pre-trap era). Of course, given appropriate values of

B_j and the country's factor endowments (k, h) prevailing, it is not difficult to satisfy the middle income condition (say, at least US$10,000 in 2005 constant PPP prices). That is, our model may support more narrowly defined middle income trap.

Finally, we note that it is possible for a country to experience multiple traps at different points of time and that technology downgrading may arise when TFP falls and physical capital decumulates. We shall illustrate all such possibilities in the quantitative analysis to which we now turn.

5 Generalization and Applications

Before conducting quantitative analysis, we would like to note that adequate generalization is needed in order to apply the bare-bone theoretical model to the real world. In this section, we will discuss how to generalize the setup, modifying the conditions for flying geese and middle income trap paradigms and then perform quantitative analysis.

5.1 Modified conditions for flying geese and middle income trap

Whereas it has been assumed that the population of labor force L is equal to one in the theoretical analysis, the growth in labor force is observed in the actual data. Introducing it into the model, we generically obtain the law of motion of human/knowledge capital when the jth technology is adopted as follows:

$$h_t = \frac{\alpha\delta\theta_j}{n_{t+1}}(h_{t-1} - \eta_j)^\sigma \quad \text{if} \quad v_{j-1} \leq h_{t-1} < v_j, \tag{47}$$

where $n_{t+1} := L_{t+1}/L_t$. The derivation of Eq. (47) is demonstrated in the Appendix.

If Assumption 2 is actually satisfied and there is no population growth of workers, the condition for trap at the jth technology is given by Eqs. (41) and (42) with $J_1 = \{j\}$. However, the calibrated pairs of θ and η do not necessarily satisfy Assumption 2. There are various patterns of the calibrated θ and η that lead a country to traps. Technically speaking, if the condition for flying geese growth does not hold in switching technologies, it is highly likely that a country falls in a trap. We consider this in the following.

Suppose that a country switches technologies from technology j to technology $j+1$ at period $T+1$. In this case, from Eq. (47), the transitional dynamics of h_t from T to $T+2$ is given by

$$\begin{cases} h_T = \frac{\alpha\delta\theta_j}{n_{T+1}}(h_{T-1} - \eta_j)^\sigma \\ h_{T+1} = \frac{\alpha\delta\theta_{j+1}}{n_{T+2}}(h_T - \eta_{j+1})^\sigma. \end{cases} \tag{48}$$

To investigate the flying-geese condition, we define functions such that $f_j(x) := (\alpha\delta\theta_j/n_{T+1})(x - \eta_j)^\sigma$ and $f_{j+1}(x) := (\alpha\delta\theta_{j+1}/n_{T+2})(x - \eta_{j+1})^\sigma$, which are the right-hand sides of (48). We also define z that denotes a potential intersection of the transition equations in (48) such that $f_j(z) = f_{j+1}(z)$ or equivalently

$$z := \frac{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\eta_j}{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}}. \tag{49}$$

Furthermore, we define z_j and z_{j+1} such that $f'_j(z_j) = f'_{j+1}(z_{j+1}) = 1$ or equivalently

$$z_j := \eta_j + \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}} \tag{50}$$

and

$$z_{j+1} := \eta_{j+1} + \left(\frac{\alpha\delta\sigma\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \tag{51}$$

The flying-geese condition is categorized into three patterns as illustrated in Figure 6. In the first case, whereas it holds that $\eta_{j+1} > \eta_j$ as in Assumption 2, it may not necessarily hold that $\theta_{j+1} > \theta_j$. In the second (the third case), it holds (may hold) that $\eta_{j+1} < \eta_j$, which contradicts Assumption 2.

First case

It follows from Figure 6 that the parameter condition for the first case is given by $\eta_j < \eta_{j+1} < z$ and $f_j(z) > z$. From Eq. (49), these inequalities can be transformed into

$$\begin{cases} \eta_j < \eta_{j+1} \\ \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j < \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right) \left(\frac{\theta_{j+1}}{n_{T+2}}\right) (\eta_{j+1} - \eta_j)^{\sigma} \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}. \end{cases} \tag{52}$$

Second case

As illustrated in Figure 6, the second flying-geese condition is given by $\eta_{j+1} < \eta_j < z$, $f_j(z) < z$, and $f_{j+1}(z_{j+1}) > z_{j+1}$. Eqs. (49)-(51) rewrite these inequalities as follows:

$$\begin{cases} \eta_{j+1} < \eta_j \\ \frac{\theta_{j+1}}{n_{T+2}} < \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} > \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right) \left(\frac{\theta_{j+1}}{n_{T+2}}\right) (\eta_j - \eta_{j+1})^{\sigma} \left[\left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}} (1-\sigma) \left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{53}$$

Third case

We obtain the necessary condition for the third case such that (i) $\eta_{j+1} < \eta_j$ and $z < \eta_j$, which is consistent with $f_{j+1}(x)$'s solid line in Figure 6, or (ii) $\eta_{j+1} > \eta_j$ and $z > \eta_j$, which is consistent with $f_{j+1}(x)$'s dotted line, ignoring the case in which $\eta_{j+1} = \eta_j$. The conditions (i) and (ii) are unified as $(\eta_{j+1} - \eta_j)(z - \eta_j) > 0$. In addition to this condition, $z < z_j$, and $z_{j+1} < f(z_{j+1})$ are essential for the third case. Eqs. (49)-(51) rewrite these inequalities as follows:

$$\begin{cases} \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}} \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} (\eta_{j+1} - \eta_j) < \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}} \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right] \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{54}$$

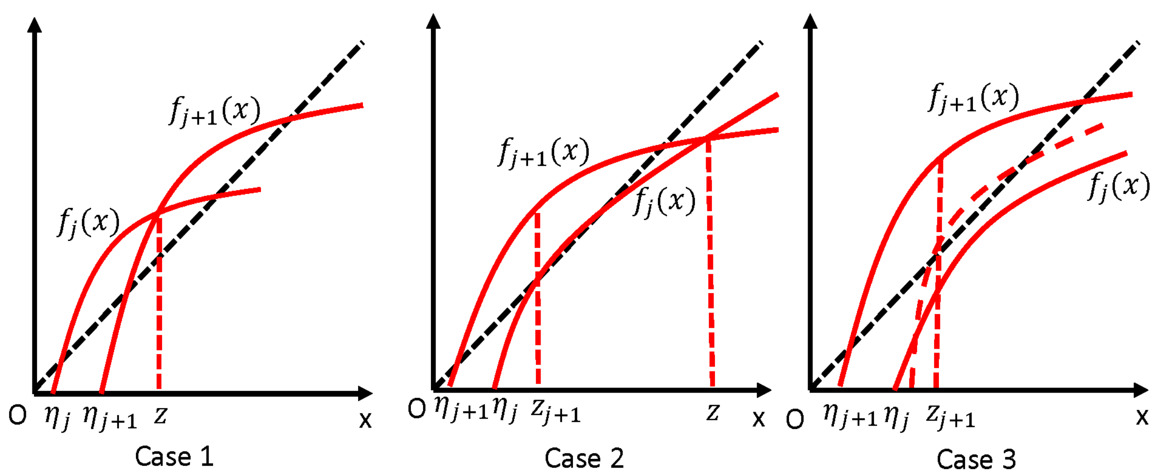

Figure 6. Flying geese condition

5.1.1 Summary

Thus, it is said that a country experiences the flying geese if and only if inequalities (52), (53), or (54) hold. If inequalities (52), (53), and (54) all fail to hold and if human capital technology fails to advance (θ stagnating and/or η rising too much relative to the technology prevailed) and/or the TFP is stalled (the TFP falling), it is said that a country falls in a middle income trap.[4] Again, the terminology of middle income is used more generally for any level of development before reaching a perpetually growing balanced growth path (that is, for all $m < M$).

[4]We will relegate the detailed criteria for a middle income trap to section 5.3.2.

5.2 Quantitative analysis

To begin, we illustrate the basis of our country selection. First, we consider four groups based on development patterns: (i) advanced countries, (ii) fast growing economies, (iii) emerging economies with decent development speed and (iv) development laggards with frequent growth slowdowns despite earlier development. Second, countries chosen are with long time series starting no later than early 1960s. Third, all countries are qualified as middle-low, middle-high or high income countries by mid-1980s (the middle year of our sample). Finally, we try to balance between continents, namely, Asia, Europe, North America and South America. The representative 14 countries from the four groups are as follows (see the Appendix for a summary of key average growth rates of each country):

(i) advanced countries: the U.S., the U.K., Germany, and Japan, with relatively stable growth ranging from 1.7% per year (the U.S.) to 3.6% (Japan);

(ii) fast growing economies: Hong Kong, S. Korea, Singapore, Taiwan, the so-called Asian Tigers, all growing at more than double of the speed of the U.S., ranging from 3.6% (Singapore) to 4.9% (Taiwan);

(iii) emerging growing economies: China, Greece, and Malaysia, all growing faster than 3.1% but slower than fast growing economies;

(iv) development laggards: Argentina, Mexico, and Philippines, featuring chopped growth paths with coefficient of variation all exceeding one and at slower speed than emerging growing economies.

As also observed by Wang, Wong and Yip (2018), most countries have decent physical capital growth but sluggish year-of-schooling-based human capital growth. Moreover, regarding the "standard" TFP, while advanced and fast growing countries have experienced decent TFP growth no lower than 1.1% annually, the TFP growth in the three development laggards have been mediocre at rates below 0.87%.

To apply the theoretical model to the real world by means of calibration, we allow the TFP, A, in Eq. (2) to be time-variant such that $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$. Define $\tilde{A}_t := A_t h_{t-1}^{\alpha}$. Then, the production function becomes $y_t = \tilde{A}_t k_{t-1}^{\beta}$. Once we obtain the data for y_t and k_{t-1} with β being fixed, $\tilde{A}_t = y_t / k_{t-1}^{\beta}$ can be computed.[5] As noted from the formation of $\tilde{A}_t$, it is composed of the TFP, A_t, and human/knowledge capital, h_{t-1}. We assume that the quality of institution affects the TFP in our analysis. Then, $A_t^{1/\alpha}$ is assumed to be a function of the institutional quality index, d_t, such that $A_t^{1/\alpha} = exp(\lambda d_t)$. As explained in the next section, the data for the institutional quality index are quoted from the Economic Freedom of the World 2019. Additionally, human/knowledge capital is assumed to be a function of the human capital (per capita) index, $\hat{h}_{t-1}$, such that $h_{t-1} = \sigma(\hat{h}_{t-1})$, the data of which are quoted form the Penn World Table, version 9.0 (PWT9.0), again as explained in the next section.[6] Accordingly, $\tilde{A}_t := A_t h_{t-1}^{\alpha}$ can be rewritten as

$$\ln \tilde{A}_t^{\frac{1}{\alpha}} = \tilde{\sigma}(\hat{h}_{t-1}) + \lambda d_t, \tag{55}$$

where $\tilde{\sigma}(\hat{h}_{t-1}) = \ln \sigma(\hat{h}_{t-1})$. We estimate Eq. (55) with $\alpha = \beta = \gamma = 1/3$ (Mankiw, Romer and Weil 1992) by a semi-parametric method.

5.2.1 Data

We draw all the data from PWT9.0 (Feenstra et al. 2015), except the institutional quality index. We assemble the annual data of the 14 countries over the period 1950–2014 although starting year varies a bit based on data availability. To obtain the per worker output, y, and the per worker physical capital, k, we use the real GDP at constant 2011 national prices (rgdpna), the capital stock at constant 2011 national prices (rkna), and the number of persons engaged (emp) in PWT9.0. We compute $\tilde{A}_t = y_t / k_{t-1}^{\beta}$ from the per worker output and the per worker physical capital. For $\hat{h}$, we use the human capital index, based on years of schooling and returns to education (hc) in PWT9.0. To

[5] One notes that $\tilde{A}_t$ is the TFP in the "standard" growth accounting.

[6] We assume that these functional forms are stationary throughout the periods to be analyzed and no other time-variant endogenous variables affect these relationships.

eliminate short-run movements, we take three-year moving average of each variable.

The data on the index of institutional quality, d, are based on the Economic Freedom of the World 2019 (Gwartney et al. 2019). Specifically, we use the index of the Economic Freedom of the World (EFW), which is a comprehensive measure of the consistency of institutions and policies with economic freedom in a country. The EFW index consists of five dimensions: (i) Size of Government, (ii) Legal System and Property Rights, (iii) Sound Money, (iv) Freedom to Trade Internationally, and (v) Regulation of credit, labor and business. The index data are available in five-year intervals from 1950 to 2000 and in annual frequency from 2000 to 2017.[7]

Table 2 shows λ estimated from Eq. (55) with a semi-parametric method for each country. In all cases, λ is positive and significant at the conventional significance level, which means that the improvement of institutional quality enhances the TFP. Once λ is estimated, $A_t = exp(\alpha\lambda d_t)$ is obtained, and accordingly, we have human/knowledge capital such that $h_{t-1} = [\tilde{A}_t / exp(\alpha\lambda d_t)]^{1/\alpha}$, which is used for the calibration of human capital technology parameters.

5.2.2 Calibrating human capital technology parameters

Suppose that the jth technology is adopted from period t to period $t+1$. Then, we can update Eq. (47) by one period and obtain

$$h_{t+1} = \frac{\alpha\delta\theta_j}{n_{t+2}} \left(h_t - \eta_j\right)^{\sigma}. \tag{56}$$

It is standard in growth models to set $\delta = 0.95$. Regarding σ, we consider three plausible values: $\sigma \in \{0.4, 0.5, 0.6\}$, and choose one to yield best fit to actual output. For a given value of σ, we apply human/knowledge capital obtained in the previous subsection to Eqs. (47) and (56) to calibrate η_j and θ_j. Accordingly, we have two equations with respect to two unknowns: η_j and θ_j. However, we note that, when

[7] The index from 1950 to 1965 does not reflect the regulation dimension. See Gwartney et al. (2019) for the detail.

Table 2. Estimation of λ

Countries	λ	Standard error	P-value
(i) Advaned Countries			
Germany	0.1502	0.0332	0.000
Japan	0.1120	0.0501	0.029
U.K.	0.0451	0.0224	0.049
U.S.	0.0483	0.0166	0.005
(ii) Fast Growing Economics			
Hong Kong	0.1487	0.0718	0.043
S. Korea	0.5511	0.1275	0.000
Singapore	0.3373	0.1586	0.038
Taiwan	0.1606	0.0340	0.000
(iii) Emerging Growing Economies			
China	0.2437	0.0721	0.001
Greece	0.2157	0.0536	0.000
Malaysia	0.0696	0.0333	0.041
(iv) Development Laggards			
Argentina	0.0716	0.0347	0.043
Mexico	0.1226	0.0439	0.007
Philippines	0.0292	0.0149	0.063

Notes. λ is estimated from Eq. (55) with a semi-parametric method for each country.

computing η_j by applying human/knowledge capital, one must set its minimum at 0. Additionally, if the computed value of η_j becomes such that $h_{t-1}-\eta_j \leq 0$ when computing θ_j, we set $\theta_j = \theta_{j-1}$, i.e., no upgrading. More concretely, the calibration procedure is as follows:

- By using Eqs. (47) and (56) and taking into account η_j's minimum setting, we compute

$$\tilde{\eta}_j = \max\left\{ \frac{n_{t+2}^{\frac{1}{\sigma}} h_{t+1}^{\frac{1}{\sigma}} h_{t-1} - n_{t+1}^{\frac{1}{\sigma}} h_t^{\frac{1+\sigma}{\sigma}}}{n_{t+2}^{\frac{1}{\sigma}} h_{t+1}^{\frac{1}{\sigma}} - n_{t+1}^{\frac{1}{\sigma}} h_t^{\frac{1}{\sigma}}}, 0 \right\}. \tag{57}$$

- If $h_{t-1} - \tilde{\eta}_j > 0$, we set $\eta_j = \tilde{\eta}_j$ and compute θ_j by using Eq. (47) as follows:

$$\theta_j = \frac{n_{t+1} h_t}{\alpha \delta \left(h_{t-1} - \tilde{\eta}_j\right)^{\sigma}}. \tag{58}$$

- If $h_{t-1} - \tilde{\eta}_j \leq 0$, we set $\theta_j = \theta_{j-1}$ and compute η_j by using Eq. (47) and taking into account η_j's minimum setting as follows:

$$\eta_j = \max\left\{ h_{t-1} - \left(\frac{n_{t+1} h_t}{\alpha \delta \theta_{j-1}}\right)^{\frac{1}{\sigma}}, 0 \right\}. \tag{59}$$

Two remarks are in order. First, although in calibrating the values of $\{\theta_j, \eta_j\}$, we have incorporated the information from the TFP and the formation of human capital, we have not accounted from any sources of slowdown due to capital or trade barriers. As a result, we, on the one hand, may miss some capital or trade-led traps and, on the other, may find that in some of our human capital-based traps, output may not grow slowly. Second, in practice, outputs fluctuate, partly driven by short-run shocks which are unrelated to the consideration of middle income traps. Thus, some smoothing strategies must be adopted. To do so, as explained in the previous section, we first take three-year moving average of each of time series to remove uninteresting short-run movements. We then compute the values of $\{\theta_j, \eta_j\}$ in non-overlapped five-year intervals (for brevity, called episodes) for each of the 14 economies, assuming that an economy employs a certain technology for each five-year interval. Because the data of human/knowledge capital for three sequential years are necessary to compute one set of $\{\theta_j, \eta_j\}$, we can have three sets of $\{\theta_j, \eta_j\}$ for each non-overlapped five-year episode. Hence, we choose a best fit to the time series of human/knowledge capital among the three sets of $\{\theta_j, \eta_j\}$ for each five-year episode by solving the following minimization problem:

$$\left\{\theta_j^*, \eta_j^*\right\} = \arg\min_{\{\theta_j, \eta_j\}} \sum_{s=0}^{4} \left| h_{t'+s+1} - \frac{\alpha\delta\theta_j}{n_{t'+s+2}} \left(h_{t'+s} - \eta_j\right)^{\sigma} \right|,$$

where t' is a starting year of a certain episode.

5.2.3 Fitness

From Eq. (47) and $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$, it follows that

$$\ln y_t = \alpha \ln(\alpha\delta) + \beta \ln k_{t-1} - \alpha \ln n_t + \ln A_t + \alpha \ln \theta_j^* + \alpha\sigma \ln(h_{t-2} - \eta_j^*). \tag{60}$$

By using calibrated A, h, θ_j^*, and η_j^* and actual k and n, the fitted value of the right-hand side of Eq. (60) can be obtained. Table 3 provides the coefficients of determination for the cases of $\sigma = 0.4$, 0.5, and 0.6. As previously explained, the best fit one is chosen for the following analysis (marked in boldface in Table 3). As noted from the table, all the coefficients of determination except that of Argentina are greater than 0.99, whereas that of Argentina is still high at 0.969. Thus, the fitness of the simulated time series of $\ln y$ is viewed good. Figures 7-10 plot the time series of the fitted and actual values of $\ln y$ for the 14 countries when a best fit σ is given in each country.

5.2.4 Flying or trapped: a unified quantitative framework

Our primary task is to identify potential traps and prolong periods of flying geese in the first three categories: advanced countries, fast growing economies, and emerging growing economies. For the cases of traps, we will also pin down the drivers of traps, possibly due to upgrading technology slowdown, technology barriers rising, or TFP

Table 3. Determination of σ

Countries	σ	R^2	Countries	σ	R^2
(i) Advaned Countries			**(ii) Fast Growing Economies**		
Germany	$\sigma = 0.4$	0.992	**Hong Kong**	$\boldsymbol{\sigma = 0.4}$	**0.998833**
	$\boldsymbol{\sigma = 0.5}$	**0.993**		$\sigma = 0.5$	0.99880
	$\sigma = 0.6$	0.992		$\sigma = 0.6$	0.9986
Japan	$\sigma = 0.4$	0.99970	**S. Korea**	$\boldsymbol{\sigma = 0.4}$	**0.9985**
	$\boldsymbol{\sigma = 0.5}$	**0.99972**		$\sigma = 0.5$	0.9984
	$\sigma = 0.6$	0.99971		$\sigma = 0.6$	0.9972
U.K.	$\sigma = 0.4$	0.99943	**Singapore**	$\sigma = 0.4$	0.9982
	$\boldsymbol{\sigma = 0.5}$	**0.99944**		$\boldsymbol{\sigma = 0.5}$	**0.9983**
	$\sigma = 0.6$	0.99942		$\sigma = 0.6$	0.9982
U.S.	$\sigma = 0.4$	0.9991	**Taiwan**	$\sigma = 0.4$	0.99952
	$\sigma = 0.5$	0.9991		$\sigma = 0.5$	0.99955
	$\boldsymbol{\sigma = 0.6}$	**0.9992**		$\boldsymbol{\sigma = 0.6}$	**0.99957**
(iii) Emerging Growing Economies			**(iv) Development Laggards**		
China	$\sigma = 0.4$	0.99566	**Argentina**	$\boldsymbol{\sigma = 0.4}$	**0.9699**
	$\boldsymbol{\sigma = 0.5}$	**0.99567**		$\sigma = 0.5$	0.9623
	$\sigma = 0.6$	0.9954		$\sigma = 0.6$	0.9673
Greece	$\sigma = 0.4$	0.9978	**Mexico**	$\boldsymbol{\sigma = 0.4}$	**0.9955**
	$\sigma = 0.5$	0.9978		$\sigma = 0.5$	0.9953
	$\boldsymbol{\sigma = 0.6}$	**0.9979**		$\sigma = 0.6$	0.9941
Malaysia	$\sigma = 0.4$	0.9992	**Philippines**	$\sigma = 0.4$	0.9909
	$\boldsymbol{\sigma = 0.5}$	**0.9993**		$\sigma = 0.5$	0.9918
	$\sigma = 0.6$	0.9992		$\boldsymbol{\sigma = 0.6}$	**0.9925**

Notes. The coefficients of determination regarding the fitted value of the right-hand side of Eq. (60) are provided for the cases of $\sigma = 0.4$, 0.5, and 0.6. The best fitted one (the boldfaced type) is chosen among them.

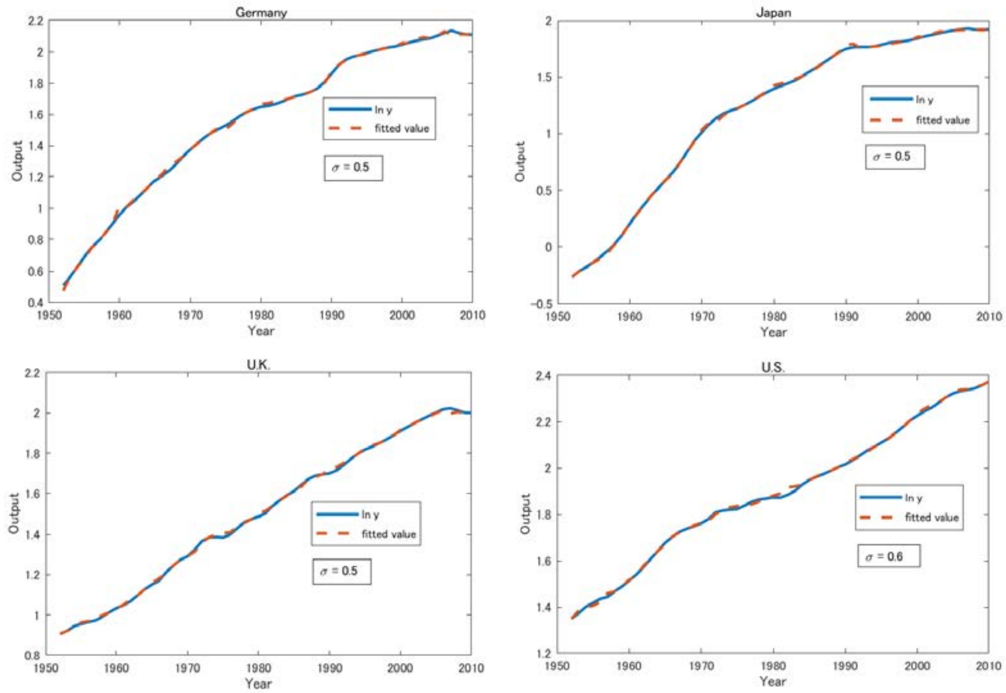

Figure 7. Output and fitted value (advanced countries)

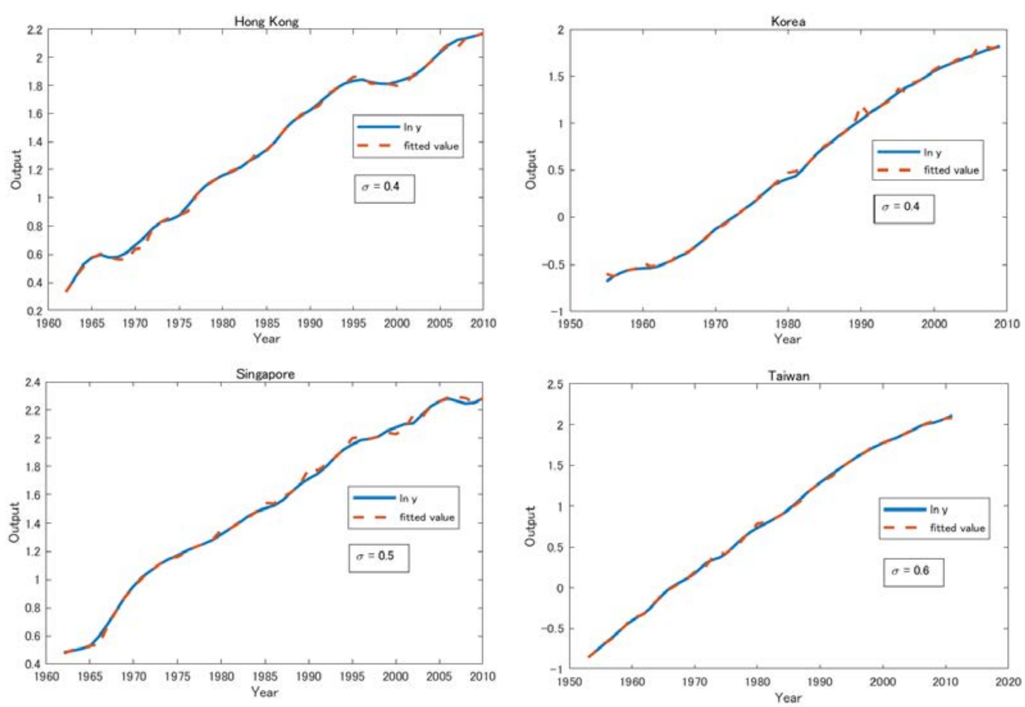

Figure 8. Output and fitted value (fast growing economies)

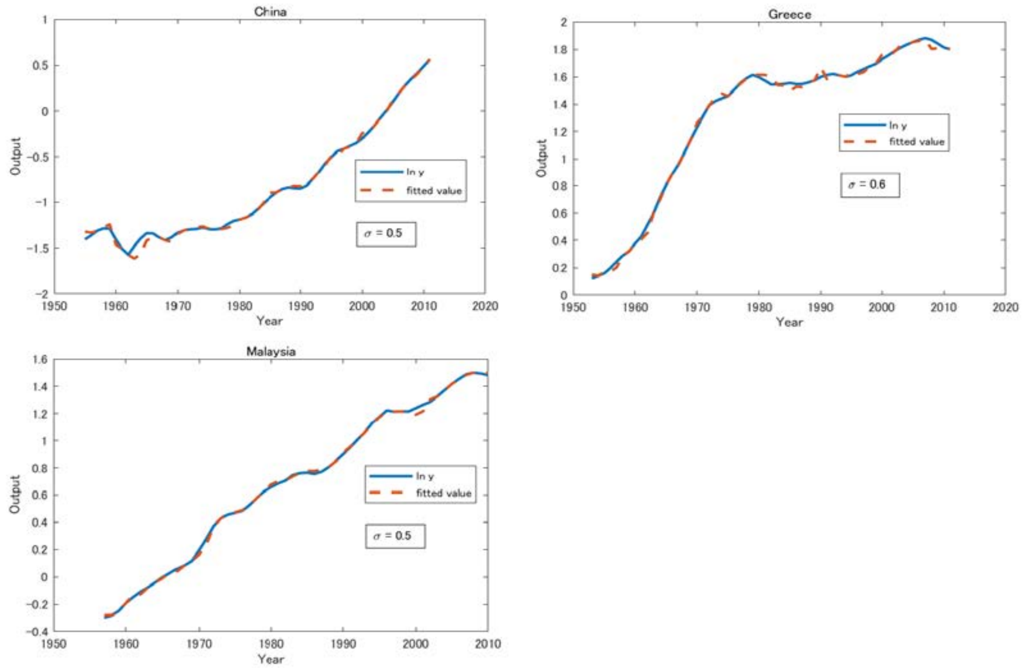

Figure 9. Output and fitted value (emerging growing economies)

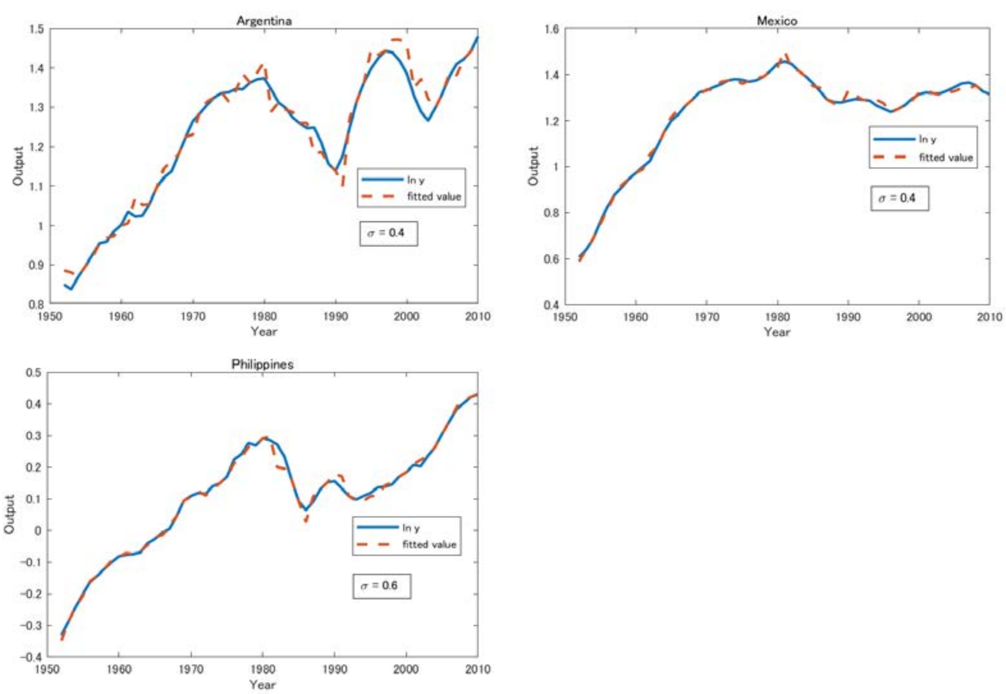

Figure 10. Output and fitted value (development laggards)

slowdown. This enables us to compare our findings with those in Eichengreen, Park and Shin (2013) where traps are identified by empirical structural breaks and with those in Wang, Wong and Yip (2018) where traps are caused by factor endowment reversal under mismatch in technology assimilation. Of course, due to different samples of countries, such comparison would be restricted to common samples only. Tables 4-7 summarize our quantitative results of advanced countries, fast growing economies, emerging growing economies, and development laggards, respectively. If either one of inequalities (52), (53), or (54) is satisfied in a period, we assign "1" to that period and judge that the country experiences flying (denoted by $\nearrow\!\!\!\!\nearrow$). If inequalities (52), (53), and (54) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then "-1" is assigned and we diagnose that the country falls in a middle income trap (denoted by $\searrow\!\!\!\!\searrow$). Otherwise, "0" is assigned, which means that the country may not experience flying but not necessarily trapped (denoted by $\Leftrightarrow$).

It is clearly seen in Tables 4 and 5 that most of the advanced countries and fast growing economies have experienced prolonged periods of flying geese paradigm with lower frequencies in traps with an exception of Japan. In these two groups of countries, Eichengreen, Park and Shin (2013) identify several middle income traps in the cases of Hong Kong in 1993, Japan in 1974, Korea in 1989, Singapore in 1980, Taiwan in 1995, and U.K. in 1988 and 2002. Wang, Wong and Yip (2018) find traps in the cases of Hong Kong in 1984 and Taiwan in 1999. Different from their works, we do not restrict the sample to fall into the World Bank ranges of middle income group. By comparison, we learn that:

- Hong Kong: flying in early 1980s and early 1990s (inconsistent with Eichengreen et al. and Wang et al.), but trapped in late 1990s and early 2000s around the Asian financial crises;

- Japan: trapped in early 1970s (consistent with Eichengreen et al.), and most certainly trapped in early 1980s during the second oil crisis;

- Korea: trapped in early 1990s (not far from 1989 identified by Eichengreen et al.) and early 2000s around the Asian financial crises;

- Singapore: trapped in early 1980s (consistent with Eichengreen et al.), and certainly trapped in early 2000s around the Asian financial crises;

- Taiwan: flying in 1990s (inconsistent with Eichengreen et al. and Wang et al.), but trapped in late 2000s during the Great Recession (which is beyond the sample period used by Eichengreen et al.);

- U.K.: trapped in late 1980s (consistent with Eichengreen et al.) and late 2000s during the Great Recession (again, which is beyond the sample period used by Eichengreen et al.).

Turning to the third group where Greece is the only sample in common, both Eichengreen et al. and Wang et al. identify a single trap in 1972, while traps from late 1970s to late 1980s are observed in our case. Relative to the first two groups, more traps are identified in the third group. For example, there are earlier traps in China through 1960s and in late 1970s when its human capital upgrading was stalled by the Great Leap Forward policy and the Cultural Revolution. Finally, moving to the last group with Mexico as the only common sample, five traps are identified in our paper. Although a trap in 1981 is identified by Eichengreen et al., a trap in late 1980s is found in our paper. We find four traps in Argentina and eight traps in Philippines.

To gain further insight, we summarize the appearance of flying, inconclusive and trapped episodes in each country in Table 8. We find that advanced and fast growing economies experienced flying paradigm more than 70% of the time while being trapped only in 25% of these episodes. On the contrary, although development laggards experienced 44% flying geese but suffered 50% of the time in trapped paradigm. For illustrative purposes, let us construct a development score by assigning 1 to each of the

Table 4. Flying or trapped (Advanced Countries)

	late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
Germany $\sigma = 0.5$	1	1	1	1	1	1	1	-1	1	1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$
Japan $\sigma = 0.5$	1	1	1	-1	-1	-1	1	-1	-1	1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$
U.K. $\sigma = 0.5$	1	1	1	1	1	1	-1	1	1	1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$
U.S. $\sigma = 0.6$	1	1	-1	1	1	1	1	1	0	1	1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Leftrightarrow$	$\Nearrow$	$\Nearrow$

Notes. If either one of inequalities (52), (53), or (54) is satisfied in a period, we assign "1" to that period judging that the country experiences flying with $\Nearrow$ denoting it. If inequalities (52), (53), and (54) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then we assign "-1" diagnosing that the country falls in a middle income trap with $\Searrow$ denoting it. Otherwise, we assign "0" meaning that the country may not experience flying but not necessarily trapped with $\Leftrightarrow$ denoting it.

Table 5. Flying or trapped (Fast Growing Economies)

	late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
Hong Kong $\sigma = 0.4$			-1	1	1	1	1	1	-1	-1	1
flying or trapped			$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$
S. Korea $\sigma = 0.4$		1	1	1	1	1	1	-1	1	-1	1
flying or trapped		$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Nearrow$
Singapore $\sigma = 0.5$			-1	1	1	-1	1	1	1	-1	1
flying or trapped			$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$
Taiwan $\sigma = 0.6$	1	1	1	1	1	1	1	1	1	1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$

Notes. If either one of inequalities (52), (53), or (54) is satisfied in a period, we assign "1" to that period judging that the country experiences flying with $\Nearrow$ denoting it. If inequalities (52), (53), and (54) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then we assign "-1" diagnosing that the country falls in a middle income trap with $\Searrow$ denoting it. Otherwise, we assign "0", meaning that the country may not experience flying but not necessarily trapped with $\Leftrightarrow$ denoting it.

Table 6. Flying or trapped (Emerging Growing Economies)

	late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
China $\sigma = 0.5$		-1	-1	1	-1	-1	-1	1	1	1	1
flying or trapped		$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$
Greece $\sigma = 0.6$	1	1	1	1	-1	-1	-1	1	1	1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$
Malaysia $\sigma = 0.5$		1	1	1	1	-1	1	1	1	-1	-1
flying or trapped		$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$

Notes. If either one of inequalities (52), (53), or (54) is satisfied in a period, we assign "1" to that period judging that the country experiences flying with $\Nearrow$ denoting it. If inequalities (52), (53), and (54) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then we assign "-1" diagnosing that the country falls in a middle income trap with $\Searrow$ denoting it. Otherwise, we assign "0" meaning that the country may not experience flying but not necessarily trapped with $\Leftrightarrow$ denoting it.

Table 7. Flying or trapped (Development Laggards)

	late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
Argentina $\sigma = 0.4$	1	1	-1	-1	1	1	-1	1	1	-1	1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$
Mexico $\sigma = 0.4$	1	1	-1	1	1	1	-1	-1	1	-1	-1
flying or trapped	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$
Philippines $\sigma = 0.6$	-1	-1	1	-1	-1	-1	1	-1	-1	0	-1
flying or trapped	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Leftrightarrow$	$\Searrow$

Notes. If either one of inequalities (52), (53), or (54) is satisfied in a period, we assign "1" to that period judging that the country experiences flying with $\Nearrow$ denoting it. If inequalities (52), (53), and (54) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then we assign "-1" diagnosing that the country falls in a middle income trap with $\Searrow$ denoting it. Otherwise, we assign "0", meaning that the country may not experience flying but not necessarily trapped with $\Leftrightarrow$ denoting it.

flying episodes, 0 to inconclusive and -1 to trapped as indicated in Table 8. We can thus compute the average development scores for each country and the group average: by definition, the score must fall between -1 and 1. Our results suggest that advanced and fast growing economies have a decent average score greater than 0.477 whereas development laggards feature an average score of -0.059.

One may then inquire what are the main reasons behind the above results. In particular, in Table 9, we summarize whether (i) large technology downgrading with θ falling by more than 5%, (ii) large increase in barriers with η rising by more than 5%, or (iii) TFP slowdowns with negative TFP growth. These are significant downward deviations from their respective trends in the 14-country panel.

Overall, the results suggest that large drops in human capital technology efficacy are overwhelmingly the primary force for a country to fall into a middle income trap. Large increases in barriers to human capital accumulation are also important, playing a bigger role in emerging growing countries. In all the countries, TFP slowdowns are only occasionally important to lead to traps. By looking into more details, we gain further insights below:

- In advanced countries, slowdown in TFP plays a small role relative to large technology downgrading in the occasional traps identified.
- In fast growing economies, large drops in human capital technology efficacy are important for their explaining their occasional traps.
- In emerging growing economies, all three factors play nonnegligible roles.
- In all three development laggards, all three factors also play nonnegligible roles. While their roles are comparable in the cases of Argentina and Philippines, slowdown in TFP plays a lesser role in the case of Mexico.

Table 8. Development scores

	Flying	Inconclusive	Trapped	Development Score
(i) Advaned Countries				
Germany	9	0	2	0.636
Japan	5	0	6	-0.091
U.K.	9	0	2	0.636
U.S.	9	1	1	0.727
Average	**8.00 (72.7%)**	**0.25 (2.3%)**	**2.75 (25%)**	**0.477**
(ii) Fast Growing Economics				
Hong Kong	6	0	3	0.333
S. Korea	8	0	2	0.600
Singapore	6	0	3	0.333
Taiwan	10	0	1	0.818
Average	**7.50 (77%)**	**0.00 (0%)**	**2.25 (23%)**	**0.538**
(iii) Emerging Growing Economies				
China	5	0	5	0.000
Greece	7	0	4	0.272
Malaysia	7	0	3	0.400
Average	**6.33 (61.3%)**	**0.00(0%)**	**4.00 (38.7%)**	**0.225**
(iv) Development Laggards				
Argentina	7	0	4	0.272
Mexico	6	1	5	0.091
Philippines	2	1	8	-0.545
Average	**5.00 (44%)**	**0.67 (6.0%)**	**5.67 (50%)**	**-0.059**

Notes. As indicated in Tables 4-7, 1 is assigned to each of the flying episodes, 0 to inconclusive, and -1 to trapped, and then the average development scores are computed for each country and each group.

Table 9. Underlying drivers to a middle income trap

Drivers	large technology downgrading (θ)	large increase in barriers (η)	TFP slowdowns
(i) Advanced Countries			
	Germany: early 90s late 00s		**Germany:** late 00s
	Japan: early 70s early 80s/early 90s late 90s	**Japan:** late 70s	**Japan:** early 70s early 90s/late 00s
	U.K.: late 80s late 00s		**U.K.:** late 00s
	U.S.: late 60s		**U.S.:** late 60s
(ii) Fast Growing Economies			
	Hong Kong: late 60s late 90s	**Hong Kong:** early 00s	**Hong Kong:** late 90s
	S. Korea: early 90s early 00s		
	Singapore: early 80s late 90s	**Singapore:** early 00s	**Singapore:** late 90s
	Taiwan: late 00s		
(iii) Emerging Growing Economies			
	China: early 60s late 70s/late 80s	**China:** late 60s early 80s	**China:** early 60s
	Greece: late 70s early 80s/late 00s	**Greece:** late 80s late 00s	**Greece:** late 70s early 80s/late 00s
	Malaysia: early 80s late 00s	**Malaysia:** early 00s	**Malaysia:** late 00s
(iv) Development Laggards			
	Argentina: early 70s early 00s	**Argentina:** late 60s late 80s	**Argentina:** late 60s early 70s/early 00s
	Mexico: late 80s early 00s	**Mexico:** late 60s early 90s/late 00s	**Mexico:** late 00s
	Philippines: early 60s late 70s/early 80s early 90s/late 00s	**Philippines:** late 50s early 70s/early 80s late 90s	**Philippines:** early 50s early 70s/early 80s late 90s

Notes. We focus on (i) large technology downgrading with θ falling by more than 5%, (ii) large increase in barriers with η rising by more than 5%, and (iii) TFP slowdowns with negative TFP growth.

5.2.5 Growth accounting

To this end, we conduct growth accounting analysis to examine the long-run quantitative importance of human capital technology and barriers played in economic growth. Taking time-difference of Eq. (60) yields

$$\triangle \ln y_t = \beta \triangle \ln k_{t-1} - \alpha \triangle \ln n_t + \triangle \ln A_t + \alpha \triangle \ln \theta_j^* + \alpha\sigma \triangle \ln \left(h_{t-2} - \eta_j^* \right),$$

which can be rewritten as

$$1 = \beta \frac{\triangle \ln k_{t-1}}{\triangle \ln y_t} - \alpha \frac{\triangle \ln n_t}{\triangle \ln y_t} + \frac{\triangle \ln A_t}{\triangle \ln y_t} + \alpha \frac{\triangle \ln \theta_j^*}{\triangle \ln y_t} + \alpha\sigma \frac{\triangle \ln \left(h_{t-2} - \eta_j^* \right)}{\triangle \ln y_t}. \tag{61}$$

This is our growth accounting basis: the five components represent growth effects of capital accumulation, employment growth, TFP advancement, human capital technology upgrading, and human capital barrier reduction (conditional on human capital accumulation). All logged differences can be measured by growth rates of the variables. In our growth accounting, we consider the growth rate for thirty years to investigate a relatively longer structural growth effect of each variable. One notes from Eq. (61) that each term in the right-hand side measures a percentage contribution of each variable corresponding to a percentage change in y_t. Table 10 presents the growth accounting results of the two thirty-year episodes.[8]

In advanced countries, human/knowledge capital technology and human/knowledge capital barriers are overwhelmingly more important drivers than physical capital accumulation (and other variables) with an exception of Japan. The second thirty years in Japan represent an unusual case in which human/knowledge capital technology and human/knowledge capital barriers have negative contributions to economic growth whereas the contribution of physical capital accumulation exceeds 100%. This outcome might reflect the stagnation in the Japanese economy after the era of high economic growth.

[8] In conducting the growth accounting with Eq. (61), we have an error term as seen in the last column of Table 10. In Table 10, we choose ones that have minimum errors among the initial and last five sets of the thirty-year growth accounting.

Table 10. Growth accounting

Growth Accounting (%)	Physical Capital	Labor	TFP	Human Capital Technology	Human Capital Barriers	Error
(i) Advaned Countries						
Germany $\sigma = 0.5$						
$\Delta \ln(y) = 1.071$ (1954-1984)	44%	1%	3%	17%	34%	1%
$\Delta \ln(y) = 0.478$ (1979-2009)	44%	0%	9%	15%	32%	0%
Japan $\sigma = 0.5$						
$\Delta \ln(y) = 1.684$ (1954-1984)	62%	0%	2%	13%	23%	0%
$\Delta \ln(y) = 0.590$ (1978-2008)	114%	1%	9%	-23%	-1%	0%
UK $\sigma = 0.5$						
$\Delta \ln(y) = 0.633$ (1952-1982)	44%	2%	2%	36%	16%	0%
$\Delta \ln(y) = 0.526$ (1979-2009)	27%	0%	6%	30%	36%	1%
U.S. $\sigma = 0.6$						
$\Delta \ln(y) = 0.525$ (1954-1984)	35%	-1%	1%	32%	34%	-1%
$\Delta \ln(y) = 0.490$ (1979-2009)	32%	3%	1%	51%	13%	0%
(ii) Fast Growing Economies						
Hong Kong $\sigma = 0.4$						
$\Delta \ln(y) = 1.348$ (1963-1993)	32%	0%	3%	35%	30%	0%
$\Delta \ln(y) = 1.010$ (1980-2010)	39%	2%	0%	47%	13%	-1%
S. Korea $\sigma = 0.4$						
$\Delta \ln(y) = 1.425$ (1956-1986)	47%	-1%	12%	30%	12%	0%
$\Delta \ln(y) = 1.451$ (1978-2008)	49%	0%	27%	14%	9%	1%
Singapore $\sigma = 0.5$						
$\Delta \ln(y) = 1.369$ (1963-1993)	33%	0%	8%	38%	20%	1%
$\Delta \ln(y) = 0.969$ (1979-2009)	39%	1%	15%	32%	14%	-1%
Taiwan $\sigma = 0.6$						
$\Delta \ln(y) = 1.720$ (1953-1983)	36%	0%	10%	23%	32%	-1%
$\Delta \ln(y) = 1.356$ (1979-2009)	43%	1%	7%	16%	34%	-1%
(iii) Emerging Growing Economies						
China $\sigma = 0.5$						
$\Delta \ln(y) = 0.454$ (1957-1987)	82%	0%	20%	-5%	2%	1%
$\Delta \ln(y) = 1.730$ (1981-2011)	45%	1%	11%	14%	29%	0%
Greece $\sigma = 0.6$						
$\Delta \ln(y) = 1.350$ (1955-1985)	41%	0%	-2%	22%	39%	0%
$\Delta \ln(y) = 0.329$ (1976-2006)	51%	-1%	30%	-14%	33%	1%
Malaysia $\sigma = 0.5$						
$\Delta \ln(y) = 1.068$ (1957-1987)	32%	0%	2%	28%	39%	-1%
$\Delta \ln(y) = 0.826$ (1980-2010)	49%	0%	0%	16%	34%	1%
(iv) Development Laggards						
Argentina $\sigma = 0.4$						
$\Delta \ln(y) = 0.362$ (1955-1985)	64%	-1%	-14%	1%	50%	0%
$\Delta \ln(y) = 0.060$ (1978-2008)	98%	-4%	134%	-191%	64%	-1%
Mexico $\sigma = 0.4$						
$\Delta \ln(y) = 0.776$ (1953-1983)	27%	-1%	-3%	27%	51%	-1%
$\Delta \ln(y) = -0.088$ (1979-2009)	113%	10%	36%	-269%	10%	0%
Philippines $\sigma = 0.6$						
$\Delta \ln(y) = 0.402$ (1954-1984)	61%	0%	-2%	-10%	51%	0%
$\Delta \ln(y) = 0.153$ (1979-2009)	86%	8%	10%	0%	-4%	0%

Notes. A percentage contribution of each variable (%) is provided corresponding to $\Delta \ln(y)$. In conducting the growth accounting with Eq. (61), we have an error term; accordingly, we show two results among the initial and last five sets of the thirty-year growth accounting that have minimum errors for each country.

As seen in Figure 7, Japan experiences a similar growth process to those of the other three advanced countries; however, the main drivers for growth in Japan appear quite different from those of the other three countries. On average in advanced countries, human capital technology upgrading and human capital barrier reduction account for 51% of economic growth during the first thirty-year window and for 38% during the second.

In the fast growing economies except Korea, human/knowledge capital technology and human/knowledge capital barriers are more important drivers than other variables, as in advanced countries. Whereas Korea experiences the high rate of growth in both periods, physical capital accumulation is the main driver of growth although its growth process is similar to the other three fast growing economies (see in Figure 8). Human capital technology upgrading and human capital barrier reduction, on average in the fast growing economies, contribute to 55% and 45%, respectively, over the two thirty-year windows.

Regarding emerging growing economies, human/knowledge capital technology and human/knowledge capital barriers continue to play relatively important roles in the second thirty-year window in China, and are more important roles than physical capital (and other variables) in the first thirty-year window in Greece and in both thirty-year windows in Malaysia. Over the two thirty-year windows, on average, human capital technology upgrading and human capital barrier reduction account for 42% and 37%, respectively, of economic growth.

Among the development laggards, in the first thiry-year window, human/knowledge capital technology and human/knowledge capital barriers are more important drivers than other variables in Mexico and remain quite important in the other two countries contributing to more than 40% of their economic growth. Over the second thiry-year window when growth rates in Argentina and Mexico turn out to be too close to zero, growth accounting results in large percentage contributions. By excluding those imprecise accounting estimates, human capital technology upgrading and human capital barrier reduction on average contribute to 52% and 41% of economic growth over the two thirty-year windows.

On the whole, in more than half of the episodes (15 out of 28 thirty-year episodes in 14 countries), human capital technology upgrading and human capital barrier reduction account for more than half of economic growth. By eliminating the two imprecise accounting estimates during the second episode, human capital technology upgrading and human capital barrier reduction contribute to 51% and 37% of economic growth, averaging over the remaining 26 episodes. In short, an interesting message we have learned from the growth accounting results is that omitting the roles played by human capital technology upgrading and human capital barrier reduction could lead to biased outcomes by a noticeable margin.

6 Concluding Remarks

We have constructed a simple growth model with endogenous technology choice in human/knowledge capital accumulation in which we are able to establish a rich array of equilibrium development paradigms, including poverty trap, middle income trap and flying geese growth. We have identified the productivity of the prevailing technology, the productivity jump of technology upgrading and the hike in technology scale barrier as the key drivers for different paradigms to arise in equilibrium. Different combination of these factors in conjunction of the TFP help understand different development patterns facing different countries.

Along these lines, an interesting avenue of future research is to introduce limitation to knowledge formation. This would allow for growing over cycles rooted on human capital. As such, it would complement the R&D based theory of growth and cycles pioneered by Matsuyama (1999) and Jovanovic (2009). Another interesting line of extension is to incorporate trade into the current framework to see how the interactions between trade and global talent flows may result in different development patterns, particularly for those economies with high trade dependence. Finally, it is also interesting to consider multiple dimensions of technology choice with physical and human capital upgrading and barriers. In so doing, one may separate different sources of traps into physical capital and human capital based. Of course, to accomplish any of these would require further simplification of the basic structure and more restrictive assumptions. These are beyond the scope of the present paper and left for future research.

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Appendix

In the Appendix, we provide proofs of various Lemmas and Propositions.

Proof of Lemma 1

To prove the first claim, define $\Psi_1(h) := B_1(h) - B_2(h)$ for $h \geq \eta_2$. Then, we have

$$\Psi_1(h) = h^\sigma - \theta_2(h - \eta_2)^\sigma \tag{A.1}$$

and

$$\Psi'(h) = \sigma[h^{\sigma-1} - \theta_2(h - \eta_2)^{\sigma-1}]. \tag{A.2}$$

Since $\Psi_1'(h) < \sigma[(h-\eta_2)^{\sigma-1} - \theta_2(h-\eta_2)^{\sigma-1}] < 0$ under Assumption 2, $\Psi_1(h)$ is a decreasing function for $h \geq \eta_2$. Additionally, it follows that $\Psi_1(\eta_2) > 0$ and $\lim_{h\to\infty} \Psi_1(h) = \lim_{h\to\infty} h^\sigma(1 - \theta_2(1 - \eta_2/h)^\sigma) = -\infty$. Therefore, the first claim of Lemma 1 holds. To prove the second claim, define $\Psi_2(h) := B_2(h) - B_3(h)$ for $h \geq \eta_3$. Then, we have

$$\Psi_2(h) = \theta_2(h - \eta_2)^\sigma - \theta_3(h - \eta_3)^\sigma \tag{A.3}$$

and

$$\Psi_2'(h) = \theta_2\sigma(h - \eta_2)^{\sigma-1} - \theta_3\sigma(h - \eta_3)^{\sigma-1}. \tag{A.4}$$

Since $\Psi_2'(h) < \sigma[\theta_2(h-\eta_3)^{\sigma-1} - \theta_3(h-\eta_3)^{\sigma-1}] < 0$ under Assumption 2, $\Psi_2(h)$ is a decreasing function for $h \geq \eta_3$. Additionally, it follows that $\Psi_2(\eta_3) > 0$ and $\lim_{h\to\infty} \Psi_2(h) = \lim_{h\to\infty} \theta_2(h - \theta_2)^\sigma[1 - (\theta_3/\theta_2)[(h - \eta_3)/(h - \eta_2)]^\sigma] = -\infty$.Therefore, the second claim of Lemma 1 holds. $\square$

Proof of Proposition 1

From Lemma 1, it follows that $B_3(h_{t-1}) > B_2(h_{t-1})$ if $h_{t-1} > v_2$. Thus, if $h_{t-1} > v_2$, the third technology is preferred to the second technology. From Lemma 1, it follows that $B_2(h_{t-1}) > B_3(h_{t-1})$ if $\eta_3 \leq h_{t-1} < v_2$. Moreover, if $\eta_2 \leq h_{t-1} < \eta_3$, the third technology is not applicable. Thus, if $\eta_2 \leq h_{t-1} < v_2$, the second technology is preferred to the third technology. Likewise, from Lemma 1, it follows that $B_2(h_{t-1}) > B_1(h_{t-1})$ if $h_{t-1} > v_1$. Thus, if $h_{t-1} > v_1$, the second technology is preferred to the first technology. From Lemma 1, it follows that $B_1(h_{t-1}) > B_2(h_{t-1})$ if $\eta_2 \leq h_{t-1} < v_1$. Moreover, if $0 \leq h_{t-1} < \eta_2$, the second technology is not applicable. Thus, if $0 \leq h_{t-1} < v_1$, the first technology is preferred to the second technology. Then, we have a desired conclusion. $\square$

Proof of Proposition 3

From the convexity of $\Pi_2(h)$ and since $k^*_{2,s}$ is obtained from Eq. (24), it suffices to show the following three claims: *Claim 1*: Eq. (25) with $j = 2$ has two distinct real number solutions, *Claim 2*: $v_1 < \hat{h}_2 < v_2$, and *Claim 3*: $\Pi_2(v_1) > 0$ and $\Pi_2(v_2) > 0$.

Claim 1

The second inequality of (30) is equivalent to inequality (26) with $j = 2$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 2, it follows from inequalities (30) that $\theta_2 > 1/\sigma^\sigma$, or equivalently,

$$\frac{\theta_2^{\frac{1}{\sigma}}}{\theta_2^{\frac{1}{\sigma}} - 1} < \frac{1}{1-\sigma}. \tag{B.1}$$

From inequality (B.1) and the second inequality of (30), we obtain

$$v_1 = \frac{\theta_2^{\frac{1}{\sigma}} \eta_2}{\theta_2^{\frac{1}{\sigma}} - 1} < \sigma^{\frac{\sigma}{1-\sigma}} (\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} = \hat{h}_2. \tag{B.2}$$

Under Assumption 2, inequality (31) yields

$$\hat{h}_2 = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < (\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} = v_2. \tag{B.3}$$

Claim 2 is proven by inequalities (B.2) and (B.3).

Claim 3

From Eq. (19), it holds that $v_1 = \theta_2^{\frac{1}{\sigma}}(v_1 - \eta_2)$. Therefore, it follows that

$$\Pi_2(v_1) = v_1(v_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}). \tag{B.4}$$

Since $\eta_2 < v_1$, from Eq. (B.4) and the first inequality of (30), we obtain

$$\Pi_2(v_1) = v_1(v_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > v_1(\eta_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > 0. \tag{B.5}$$

From Claim 1, it follows that

$$\Pi_2(\hat{h}_2) < 0. \tag{B.6}$$

From inequality (31), we have $\hat{h}_2 < \eta_3$ and

$$\Pi_2(\eta_3) := \eta_3(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + (\delta\alpha\theta_2)^{\frac{1}{\sigma}}\eta_2 > 0. \tag{B.7}$$

From inequalities (B.6), (B.7), and $\eta_3 < v_2$ with the convexity of $\Pi_2(h)$, it holds that

$$\Pi_2(v_2) > 0. \tag{B.8}$$

Claim 3 is proven by inequalities (B.5) and (B.8). $\square$

Proof of Proposition 4

From the convexity of $\Pi_3(h)$ and since $k^*_{3,s}$ is obtained from Eq. (24), it suffices show the following three claims: *Claim 1*: Eq. (25) with $j = 3$ has two distinct real number solutions, *Claim 2*: $v_2 < \hat{h}_3$, and *Claim 3*: $\Pi_3(v_2) > 0$.

Claim 1

The second inequality of (32) is equivalent to inequality (26) with $j = 3$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 2, it follows from inequalities (32) that $\theta_2^{\frac{1}{\sigma}}/\theta_3^{\frac{1}{\sigma}} < \sigma$, or equivalently,

$$\frac{\theta_3^{\frac{1}{\sigma}}}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{1}{1-\sigma}. \tag{C.1}$$

From inequality (C.1) and the second inequality of (32), we obtain

$$v_2 = \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}} = \hat{h}_3, \tag{C.2}$$

which is Claim 2.

Claim 3

From Eq. (20), it holds that $\theta_2^{\frac{1}{\sigma}}(v_2 - \eta_2) = \theta_3^{\frac{1}{\sigma}}(v_2 - \eta_3)$. Therefore, it follows that

$$\Pi_3(v_2) = v_2(v_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}}. \tag{C.3}$$

Since $\eta_3 < v_2$, from Eq. (C.3) and the first inequality of (32), we obtain

$$\Pi_3(v_2) = v_2(v_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > v_2(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > 0, \tag{C.5}$$

which is Claim 3. $\square$

Proof of Lemma 3

Eq. (23) rewrites $\alpha\delta B_j'(h_{j,s}^*)$ as

$$\alpha\delta B_j'(h_{j,s}^*) = \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(h_{j,s}^*)^{\frac{1-\sigma}{\sigma}}}. \tag{D.1}$$

Since $h_{j,1}^* < \hat{h}_j = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}} < h_{j,2}^*$, the use of Eq. (D.1) yields

$$\alpha\delta B_j'(h_{j,1}^*) > \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{D.2}$$

and

$$0 < \alpha\delta B_j'(h_{j,2}^*) < \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{D.3}$$

Inequalities (D.2) and (D.3) are desired conclusions. $\square$

Proof of Proposition 6

From Eq. (23) and the definitions of v_1 and v_2, it suffices to show that $v_i < \alpha\delta\theta_{i+1}(v_i - \eta_{i+1})^\sigma$ for $i = 1, 2$. $v_i < \alpha\delta\theta_{i+1}(v_i - \eta_{i+1})^\sigma$ is equivalent to $(\alpha\delta)^{\frac{1}{1-\sigma}} > \Phi_i$. From the last inequality, we obtain the desired conclusion. $\square$

Proof of Proposition 7

For any $j \in J$, from the first inequality of (41), it follows that $\alpha\delta B_j(v_j) < v_j$. Additionally, from the second inequality of (41), it follows that $v_1 < \alpha\delta B_1(v_1)$. By technology choice, the equation for the transitional dynamics with respect to h_t (i.e., Eq. (38) extended to the case of M technologies) is continuous. Therefore, there can be more than one steady state with multiple technologies in J that feature middle income trap. $\square$

Derivation of Eq. (47)

When the growth in labor force is introduced into the model, Eqs. (4) and (5) are changed into

$$k_\tau = g_0(i_\tau^k)/n_{\tau+1} \tag{F.1}$$

and

$$h_\tau = \max_{m=1,2,\ldots,M}\{g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)\}/n_{\tau+1}, \tag{F.2}$$

where $n_{\tau+1} = L_{\tau+1}/L_\tau$ whereas Eq. (3) remains the same. Then, the first-order conditions are obtained as follows:

$$\lambda_t = \frac{1}{c_t}, \tag{F.3}$$

$$\lambda_t = \left(\frac{\delta\alpha b(\bar{h}_{t-1}, \bar{y}_t)y_{t+1}}{n_{t+1}h_t}\right)\lambda_{t+1}, \tag{F.4}$$

$$\lambda_t = \left(\frac{\delta\beta y_{t+1}}{n_{t+1}k_t}\right)\lambda_{t+1}, \tag{F.5}$$

$$n_{t+1}\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t)p_t^h. \tag{F.6}$$

The same manipulations as those in section 3 yield exactly the same equations with respect to $q_t^k = p_t^k k_t$ and $q_t^h = p_t^h h_t$ as Eqs. (15) and (16), respectively. Additionally, with respect to h_t and k_t, it follows that

$$h_t = \frac{\alpha \max_m\{B_m(h_{t-1})\}}{n_{t+1}}\left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma}\left(\frac{1}{q_{t-1}^k}\right)\right], \tag{F.7}$$

and

$$k_t = \frac{\beta A h_{t-1}^\alpha k_{t-1}^\beta}{n_{t+1}}\left[\frac{1}{1-\gamma} - \frac{\beta\delta}{1-\gamma}\left(\frac{1}{q_{t-1}^k}\right)\right]. \tag{F.8}$$

Again, from the same manipulations as those in section 4, we obtain the law of motion of human/knowledge capital when $j = \arg\max_m\{B_m(h_{t-1})\}$ as follows:

$$h_t = \frac{\alpha\delta\theta_j}{n_{t+1}}(h_{t-1} - \eta_j)^\sigma \quad \text{if} \quad v_{j-1} \le h_{t-1} < v_j$$

which is Eq. (47).

Data summary

The average growth rates of output per worker (y), physical capital per worker (k), human/knowledge capital per worker (h), and TFP (A) of each country are summarized below.

Table A.1. Average Growth Rates

Average Growth (%)	y	k	h	A	$\tilde{A}$
Advanced Countries					
Germany	2.7	3.2	4.5	0.1	1.6
Japan	3.6	7.4	3.2	0.1	1.1
U.K.	1.8	1.9	3.3	0.05	1.2
U.S.	1.7	1.7	3.4	0.01	1.1
Fast Growing Economies					
Hong Kong	3.7	3.6	7.3	0.1	2.5
S. Korea	4.3	5.6	4.1	1.0	2.4
Singapore	3.6	3.6	6.7	0.2	2.4
Taiwan	4.9	5.4	8.4	0.36	3.1
Emerging Growing Economies					
China	3.5	5.6	3.5	0.4	1.6
Greece	2.7	3.2	4.3	0.18	1.6
Malaysia	3.1	3.3	6.0	0.03	2.0
Development Laggards					
Argentina	1.1	1.4	1.8	-3.9×10^{-5}	0.63
Mexico	1.2	1.3	2.0	0.05	0.77
Philippines	1.4	1.6	2.6	0.01	0.87

Notes. Output per worker (y) and physical capital per worker (k) are directly computed by using the data of PWT9.0. Human/knowledge capital per worker (h) and TFP (A) are obtained as indicated in section 5.2. $\tilde{A}$ is the TFP in the standard growth accounting, which is computed from y and k.