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Extended Implicative Relations

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Abstract

The article explores extended implicative relations. Extended implicative relations use extended implication. Extended implication describes: relation, consequence, causation and operation. The article shows that extended implication can serve as a complexity assessment tool. The content of implicative information relations is revealed. Implicative information relations are a type of information relations. Implicative information relations describe statics and dynamics in the information field. Statics is about the relationships between information models and their parts. The dynamics of information implicative relations lie in the relationships between the inputs and outputs of information processes. The dynamics of information implicative relations lie in the relationships between the states of information situations and the states of objects in the information field. The formalism for describing implicate information relations and implicate relations is approximately the same in that case unless coordination and configuration parameters are applied. Implicative operational relations allow the assessment of procedural complexity. The difference between simple and complex implicative relations is shown. Complexity estimates for arguments and operations are shown. Taking into account the coordination and configuration of initial objects or sets allows us to expand the concept of implication and introduce the concept of “morphological implication”. Morphological implication is used to describe the transformation operations of a company. The result of morphological implication depends on the relationships between the original sets or configurations. Morphological implication is used in spatial logic. In spatial logic, the results of implicative operations are diverse, since they depend on factors that ordinary logic does not take into account.

Keywords: relations, implication, extended implication, informational implication, morphological implication, spatial logic.

1. Introduction

Currently, there is growing interest in the problems of describing and modeling complexity and information description of processes and situations. Information description is divided into descriptive and procedural. Descriptive description is associated with information models and information relationships. The procedural information description is associated with information processes and information interactions. One of the universal means of description is implication. Its peculiarity is that it can describe information relations in a state and in a situation. This is a

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static model of application of implication. Implication can describe information processes and situation dynamics. This is a dynamic model of application of implication. Implication can describe connections. This is a dynamic – static model of application of implication. An implication is denoted by a single symbol, so it can be considered an information unit. In the social sphere, implication is often identified with the term "consequence" (Sorensen et al., 1998). In logic, implication acts as a logical connective. There are implicative relations (Doran, Martin, 2021). A related implication is the concepts of "derivability" or "followability" (Visser, 2022). The development of the concept of implication is "logic of bunched implications" – BI (Gheorghiu, Marin, 2021; Gheorghiu, Pym, 2023). The propositional version of BI arises from an analysis of the evidential-theoretic relationship between conjunction and implication; it can be seen as a fusion of intuitionistic logic and multiplicative intuitionistic linear logic. The naturalness of BI can be viewed categorically: models of propositional evidence. This suggests that implication and implicative relations can be applied in the theory of evidence and in the theory of cause-and-effect analysis. An extension of the BI method is the logic of linear temporal grouping of implications (LTBI) (Galmiche, Méry, 2023). Implication is used in temporal logic and modal logic. Overall, this is a fairly universal description tool. Implication is more often applied than researched. There are few works devoted to the study of implication as a universal description mechanism. This article fills this gap.

2. Results and discussion

The variety of applications of implicative relations.

Implicative relations are usually associated with logic and used in the logical field (Baiyere et al., 2020). However, implication is used in many ways. Implication has many interpretations that complement each other. Symbolically, the implication is displayed by arrows that indicate direct (1) or reverse (2) implication

$$A \rightarrow B, C \quad (1)$$

$$D, E \leftarrow F \quad (2)$$

Expression (1) can be interpreted as follows: event A entails events B, C. Expression (2) can be interpreted as follows: event F can have events D E as a cause. This example shows that implication is a tool for cause-and-effect analysis. In the causal aspect, implication describes the causal relationship between the premises and the conclusion.

Expression (1) can be interpreted differently: set A is divided into sets B, C. Expression (2) can be interpreted as follows: set F can have D E as subsets. This example shows that implication is a tool for structural analysis.

Expression (1) can be interpreted as follows: category A is divided into subcategories B, C. Expression (2) can be interpreted as follows: category F can have D E as subcategories. This example shows that implication is a tool for categorical or qualitative analysis

An implication can express a proposition. In this case, in expression (1) A is a condition sufficient for the fulfillment of corollary B, C. Corollary B, C is a condition necessary for the truth of premise A.

Implicative information relations.

In their simplest interpretation, implicative relations describe relations of logical consequence in a logical field. Implicative information relations are a type of information relations in the information field (Tsvetkov, 2014). Implicative information relations describe statics and dynamics in the information field. Statics lies in the relationships between information models, between parts of information models, between information units, between information situations and information structures. The dynamics of information relations lies in the relationships between the inputs and outputs of information processes. The dynamics of information relations lies in the relationships between the states of information situations and the states of objects in the information field. The dynamics of implicative information relations sets cause-and-effect relationships and connections.

Implicative information relations are used individually and in groups. Single implications describe one-time processes or one-time changes in states. In a group, implicative relations form chains or sequences. Sequences (Zaheer et al., 2020) are sequences that describe: sequential change: states, operations, transformations, argumentation, conclusions.

Implicative information relations are the transfer of implicative relations into the information field or a particular example of information relations (Cross, Sproull, 2004). The formalism for describing implicate information relations and implicate relations is approximately the same in that case unless coordination and configuration parameters are applied. Implicative information relations are denoted using the implication symbol. The implication is written using an arrow as

$$A \rightarrow B.$$

In such a notation, object A is called a premise, object B is called a consequence. Implication is interpreted in different ways, for example, as a logical connective approximating the interpretation "if..., then...".

In Boolean logic, implication is considered as a function of two variables. These variables are called operands, operations or function arguments. In general, an implication describes an operation. Using the example of implicate relations, we can evaluate operational complexity. Operational complexity exists in operations research. There are simple implicative relations that consist of one implication. A simple implicative relation consists of one implication and two arguments. This implication describes one operation between the arguments M1 and M2.

$$M1 \rightarrow M2 \quad (3)$$

Expression (3) describes the succession relation between an object M1 and another object M2. Expression (3) has a multi-valued interpretation. For example, the state of an object M1 entails another state of the same object M2. Situation M1 entails another situation M2. The computational stage M1 entails another computational stage M2. These implicative relations appear in the state space. In the information field, expression (3) can describe the transformation of model M1 into model M2.

Implicative operational relationships allow complexity to be assessed. There are complex implicative relations that are divided by the number of operations and the number of arguments. Complex implicative relations based on the number of arguments are given in (1), (2). Additionally, the following examples of complex implication can be given.

$$A, C \rightarrow B \quad (4)$$

$$(D, E, G) \rightarrow A, B, C \quad (5)$$

Argument complexity (Comp(arg)) or argumentative complexity appears in implicative relations that involve functions of several arguments

$$F(A1, A2, A3, An) \rightarrow B. \quad (6)$$

In expression (6), the greater the number of arguments, the higher the argument complexity (Comp(arg)). Complexity in arguments entails ambiguity of the result (consequence).

Operational complexity (Comp(n)) appears in implicate relations, which consist of chains of simple relations.

$$A1 \rightarrow A2 \rightarrow A3 \rightarrow \ldots \rightarrow An \quad (7)$$

Expression (7) is called a chain of operations. The greater the number of operations, the higher the operational complexity. The probability of events in the chain is determined using the formula

$$P(An) = P(A1)\, P(A2) \ldots P(An\text{-}1) \quad (8)$$

From expression (8) it follows that the longer the chain, the lower the probability of the last operation. Complexity of operations reduces the reliability of the final consequence An.

Morphological implication.

Taking into account the coordination and configuration of initial objects or sets allows us to expand the concept of implication and introduce the concept of "morphological implication". Morphological implication between objects is such an implication, the result of which depends on the morphology of the objects. A morphological implication includes two or more participant objects of the implication and a result object.

Taking into account coordination and configuration is necessary in the field of spatial information, in particular in geoinformatics. Let us consider the formation of a new object as an implication of set-theoretic or spatial objects. For example:

$$M1 \cup M2 \rightarrow M3 \quad (9)$$

Expression (9) says that the sum or union of objects M1 and M2 entails the creation of object M3.

If M1 and M2 are informational or parametric sets and they are connected by a union relation, then the result of the implication (output set M3) will depend on the set-theoretic relations between them. For example, if there is an overlap between the source sets M1 and M2, then the result of the implication or output set M3 has the form shown in Figure 1.

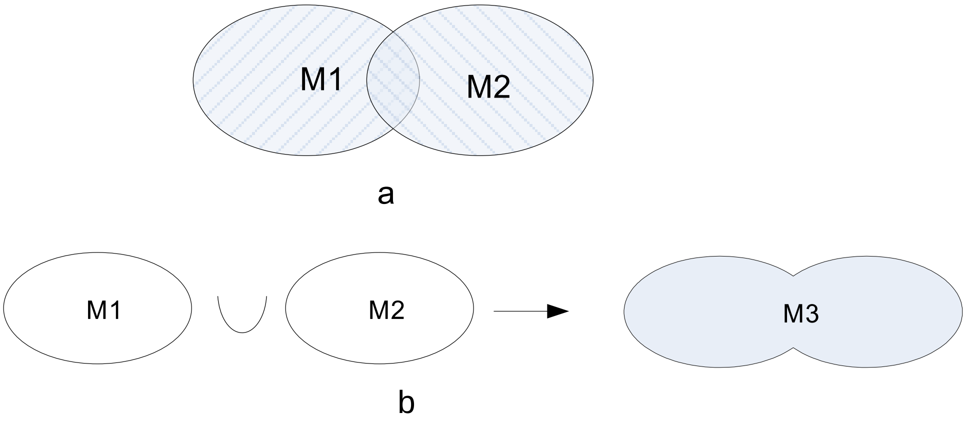

Fig. 1. Implicative relation overlap

Option "a" in Figure 1 shows the original set-theoretic relation between M1 and M2. Option "b" in Figure 1 shows the result of the implication or the output set M3. For this situation there is

$$M1 \cap M2 \neq \varnothing.$$

$$M3>M1;\ M3>M2.$$

If there is no overlap between the sets M1 and M2, but there is a tangency, then the result of the union will be different. Combining objects in the absence of overlap is shown in Figure 2.

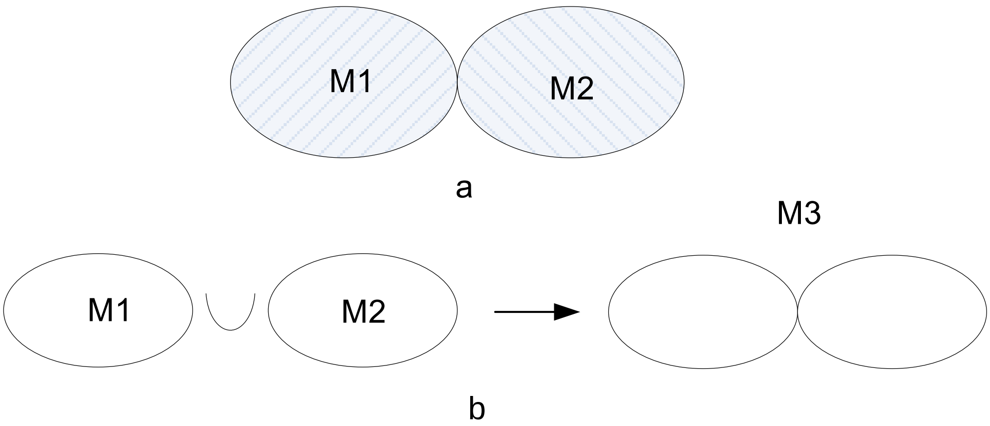

Fig. 2. Implicative relation association

Option "a" in Figure 2 shows the set-theoretic relation between M1 and M2. Option "b" in Figure 2 shows the result of the implication. For this situation there is

$$M1 \cap M2 = \varnothing;$$

$$M3=M1+M2.$$

The result of implication based on the union operation can be absorption (Figure 3).

Option "a" in Figure 3 shows the set-theoretic relation between M1 and M2. Option "b" in Figure 3 shows the result of the implication. For this situation there is

$$M1 \cup M2 \rightarrow M2=M3$$

In the set-theoretic formalism, Figure 3b corresponds to an expression that is not characteristic of arithmetic.

$$M1+M2= M2$$

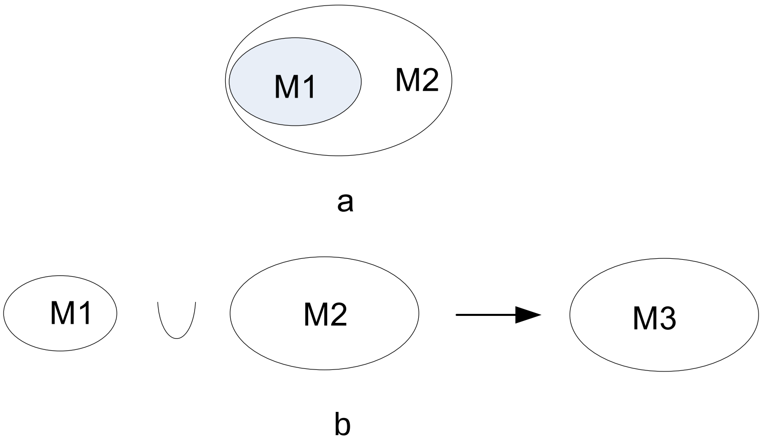

Fig. 3. Implicative relation absorption

Conclusion: the result of the implication when combining sets depends on the set-theoretic relations between the original sets M1 and M2. This feature requires the introduction of a new concept to distinguish between logical implication and set implication described in the examples given. This new concept is morphological implication. Morphological implication is an implication whose result depends on the morphology of objects and the set-theoretic relations between them. Figures 1b, 2b, 3b are examples of morphological implication. Morphological implication takes into account coordination parameters and configuration parameters. Coordination parameters are not taken into account in ordinary logic and set theory. Coordination parameters are taken into account in spatial logic. The result of morphological implication in spatial logic (Janoschka et al., 2020; Kudzh, Tsvetkov, 2020) differs for figures of different shapes (ellipses, squares, circles and bodies of arbitrary shape). In logic, the result of implication is the same, but in spatial logic the results of implicate relations are significantly different. Consequently, the formal application of the implication operator does not provide an unambiguous interpretation of the result. To apply implication in a “non-logical” sphere, additional information is needed for an unambiguous interpretation of the implicature relation. In particular, coordination information about the original sets is needed.

The information situations in Figure 1a, Figure 2a, Figure 3a show that the result of morphological implication changes depending on the type of relationship with the same functional connection between the arguments. Let us show that the result of the implication varies depending on the type of function or relations between the original sets.

Let's consider the situation in Figure 1a for another relationship between the original sets. In Figure 1, the set-theoretic relation “union” was used. Let's consider another relation "intersection". In Figure 4a the situation in Figure 1a is repeated, but a different result of the relationship is shown in Figure 4b.

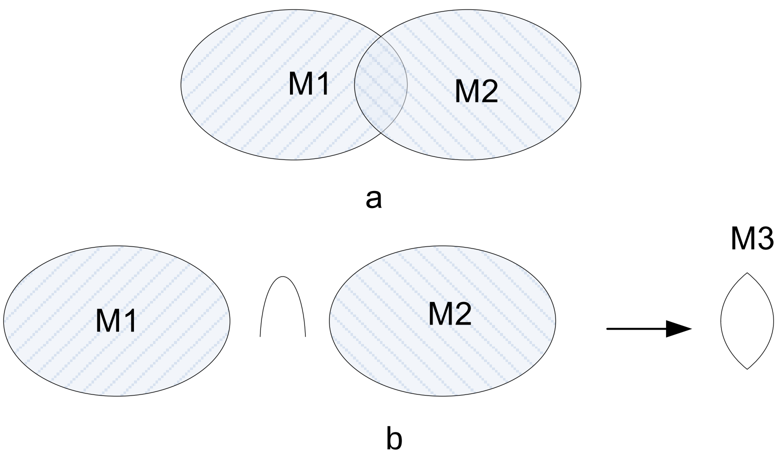

Fig. 4. Implicative relation at intersection

The implication in Figure 1 increases the output set M3 compared to the original ones. The implication in Figure 4 reduces the output set M3 compared to the original ones. Option "a" in Fig. 4 shows the set-theoretic relation between M1 and M2. Option "b" in Figure 4 shows the result of the implication. For this situation

$$M3<M1; M3<M2.$$

A comparison of Figure 1 and Figure 4 shows that the result of the implication set M3 is significantly different for the operation of union and intersection. It has a different morphology.

Configuration, together with coordination, also influences the result of implication. Figure 5 shows a situation similar to that in Figure 4, but with a different morphology of the original sets.

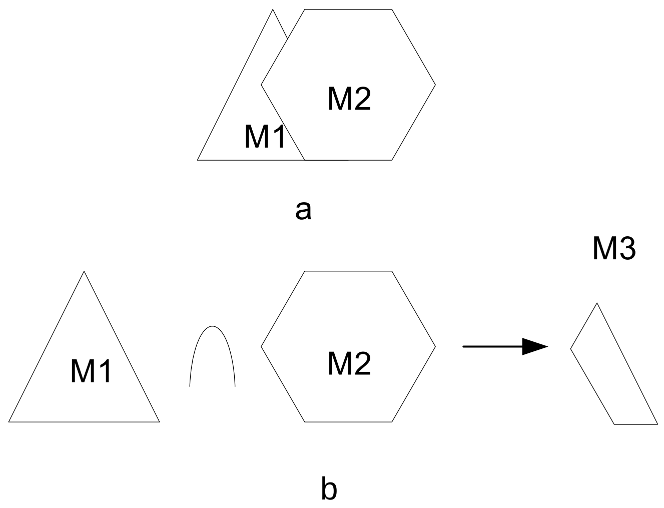

Fig. 5. Implicative relation of intersection with another morphology of original sets

Option "a" in Figure 5 shows the set-theoretic relation between M1 and M2. Option "b" in Figure 5 shows the result of the implication. A comparison of Figure 1, Figure 4 and Figure 5 shows that the result of the morphological implication of the set M3 differs significantly depending on the morphology of the original sets (arguments). Such a difference does not appear in set theory and ordinary logic. This difference is revealed by the methods of spatial logic.

Let's make another comparison. Let's take the situation in Figure 2a as the initial one, but replace the union operation with intersection. The result is shown in Figure 6.

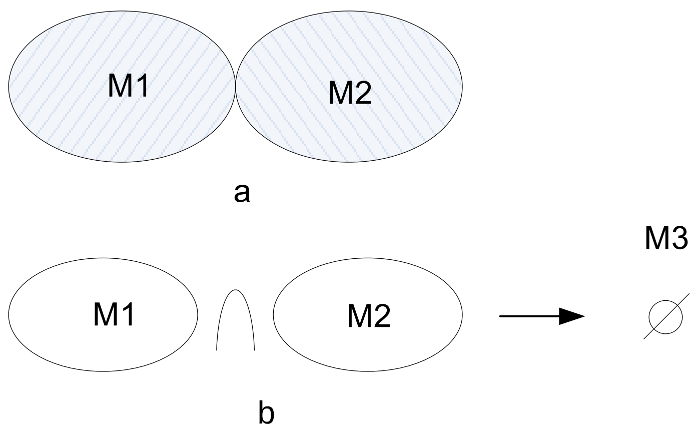

Fig. 6. Implicative relation when combining disjoint sets

Option "a" in Figure 6 shows the set-theoretic relation between M1 and M2. Option "b" in Figure 6 shows the result of the implication. The result of the implication, the set M3 in Figure 6 is an empty set, in contrast to M3 in Figure 2.

3. Conclusion

Implication can be considered as a relation, a consequence and an operation. Implication can be thought of as a cause-and-effect relationship. Morphological implication is an operation with sets, the result of which depends on set-theoretic and spatial relations between the original sets. Implication can take into account spatial relationships. In operations theory, implication can serve as a tool for assessing complexity. This complexity is related to operations and relationships. Implication can serve as a means of describing operations and processes. In combination with set theory and morphology, implication can serve as a tool of spatial logic. For this purpose, a new concept of morphological implication is introduced. Morphological implication exists in spatial logic. In spatial logic, the results of implicate relations are significantly different, since they depend on factors that ordinary logic does not take into account. In relation to information modeling, the article introduces the concept of "implicit information relations". Implicative information relations are relationships in the information field that exist between information models, between parts of information models, between information units, between information situations and information structures. Implicative relations in spatial information are multivalued and differ from implicative relations in classical logic. They are the subject of further research.

References

Baiyere et al., 2020 – *Baiyere, A., Salmela, H., Tapanainen, T.* (2020). Digital transformation and the new logics of business process management. *European journal of information systems*. 29(3): 238-259.

Cross, Sproull, 2004 – *Cross, R., Sproull, L.* (2004). More than an answer: Information relationships for actionable knowledge. *Organization science*. 15(4): 446-462.

Doran, Martin, 2021 – *Doran, Y.J., Martin, J.R.* (2021). Field relations: Understanding scientific explanations. *Teaching Science* (pp. 105-133). Routledge.

Galmiche, Méry, 2023 – *Galmiche D., Méry D.* (2023). Labelled tableaux for linear time bunched implication logic. 8th International Conference on Formal Structures for Computation and Deduction (FSCD 2023). Schloss Dagstuhl-Leibniz-Zentrum für Informatik.

Gheorghiu, Marin, 2021 – *Gheorghiu, A., Marin, S.* (2021). Focused Proof-search in the Logic of Bunched Implications. *FoSSaCS*. March, pp. 247-267.

Gheorghiu, Pym, 2023 – *Gheorghiu, A.V., Pym, D.J.* (2023). Semantical Analysis of the Logic of Bunched Implications. *Studia Logica*, 1-47.

Janoschka et al., 2020 – *Janoschka, M., Alexandri, G., Ramos, H. O., Vives-Miró, S.* (2020). Tracing the socio-spatial logics of transnational landlords' real estate investment: Blackstone in Madrid. *European urban and regional studies*. 27(2): 125-141.

Kudzh, Tsvetkov, 2020 – *Kudzh, S., Tsvetkov, V.* (2020). Spatial logic concepts. *Revista Inclusiones*, 837-849.

Sorensen et al., 1998 – *Sorensen, G., Emmons, K., Hunt, M. K., Johnston, D.* (1998). Implications of the results of community intervention trials. *Annual review of public health*. 19(1): 379-416.

Tsvetkov, 2014 – *Tsvetkov, V.Ya.* (2014). Information field. *Life Science Journal*. 11(5): 551-554.

Visser, 2022 – *Visser, F.T.* (2022). Indirect Consecution. *An Historical Syntax of the English Language* (pp. 2234-2338). Brill.

Zaheer et al., 2020 – *Zaheer, M., Guruganesh, G., Dubey, K.A., Ainslie, J., Alberti, C., Ontanon, S., ... , Ahmed, A.* (2020). Big bird: Transformers for longer sequences. *Advances in neural information processing systems*. 33: 17283-17297.