SUPPLEMENT F: DERIVATION OF THE FORM OF CONFIDENCE INTERVAL FOR THE PROPORTION OF VARIABILITY ATTRIBUTABLE TO UMPIRES

It is well known that the mean squares S_1^2, S_2^2, and S_3^2 correponding to the main effects of umpires, the interaction effects between umpires and batter handedness, and the residual error, respectively from the analysis of variance table of a fixed-effects version of model (7.1) are independent under model (7.1), and that the following distributional results hold under model (7.1):

$$\begin{aligned}
&(m-1)S_1^2/(qr\sigma_u^2 + r\sigma_{\beta u}^2 + \sigma_e^2) \sim \chi^2(m-1),\\
&(q-1)(m-1)S_2^2/(r\sigma_{\beta u}^2 + \sigma_e^2) \sim \chi^2((q-1)(m-1)),\\
&qm(r-1)S_3^2/\sigma_e^2 \sim \chi^2(qm(r-1)).
\end{aligned}$$

Here $q = 2$ is the number of levels of batter handedness, $m = 86$ is the number of umpires, and $r = 2$ is the number of replications per umpire. It is also well known that minimum variance quadratic unbiased estimators of the variance components are as follows:

$$\begin{aligned}
\widehat{\sigma_u^2} &= (S_1^2 - S_2^2)/qr,\\
\widehat{\sigma_{\beta u}^2} &= (S_2^2 - S_3^2)/r,\\
\widehat{\sigma_e^2} &= S_3^2.
\end{aligned}$$

Now define

$$\begin{aligned}
\theta_1 &= qr\sigma_u^2 + r\sigma_{\beta u}^2 + \sigma_e^2,\\
\theta_2 &= r\sigma_{\beta u}^2 + \sigma_e^2,\\
\theta_3 &= \sigma_e^2,
\end{aligned}$$

and let $\alpha \in (0,1)$. Then, applying a general result given by Lu, Graybill, and Burdick (1987), an approximate $100(1-\alpha)\%$ upper confidence interval for $[\theta_1 + (q-1)\theta_2]/\theta_3$ is $[L, \infty)$, where

$$L = \left(1 - \frac{2}{qm(r-1)}\right)\frac{S_1^2 + (q-1)S_2^2}{S_3^2} - \frac{(a_L S_1^4 + b_L(q-1)^2 S_2^4 + c_L(q-1)S_1^2 S_2^2)^{1/2}}{S_3^2}$$

with

$$\begin{aligned}
a_L &= \left[1 - \frac{2}{qm(r-1)} - F^{-1}(\alpha, m-1, qm(r-1))\right]^2,\\
b_L &= \left[1 - \frac{2}{qm(r-1)} - F^{-1}(\alpha, (q-1)(m-1), qm(r-1))\right]^2,\\
c_L &= \left[1 - \frac{2}{qm(r-1)} - F^{-1}(\alpha, q(m-1), qm(r-1))\right]^2 \frac{q^2(m-1)^2}{(q-1)(m-1)^2} - a_L/(q-1) - b_L(q-1).
\end{aligned}$$

Now observe that

$$\frac{\sigma_u^2 + \sigma_{\beta u}^2}{\sigma_e^2} = \frac{1}{qr}\frac{\theta_1 + (q-1)\theta_2}{\theta_3} - \frac{1}{r},$$

and

$$\gamma \equiv \frac{\sigma_u^2 + \sigma_{\beta u}^2}{\sigma_u^2 + \sigma_{\beta u}^2 + \sigma_e^2} = \frac{(\sigma_u^2 + \sigma_{\beta u}^2)/\sigma_e^2}{(\sigma_u^2 + \sigma_{\beta u}^2)/\sigma_e^2 + 1}.$$

It follows that

$$\begin{aligned}
1 - \alpha &= P\left(L \le \frac{\theta_1 + (q-1)\theta_2}{\theta_3} < \infty\right)\\
&= P\left(\frac{1}{qr}L - \frac{1}{r} \le \frac{\sigma_u^2 + \sigma_{\beta u}^2}{\sigma^2} < \infty\right)\\
&= P\left(\frac{\frac{1}{qr}L - \frac{1}{r}}{\frac{1}{qr}L - \frac{1}{r} + 1} \le \frac{\sigma_u^2 + \sigma_{\beta u}^2}{\sigma_u^2 + \sigma_{\beta u}^2 + \sigma_e^2} < 1\right)
\end{aligned}$$

Thus, an approximate 100(1-α)% upper confidence interval for γ is given by

$$\left[\frac{\frac{1}{qr}L - \frac{1}{r}}{\frac{1}{qr}L - \frac{1}{r} + 1}, 1\right).$$